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Trade and International Economics

International trade at the firm-to-firm level

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Part I

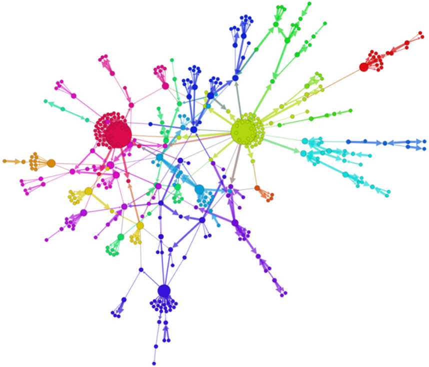
Introduction

Motivation

- Global trade is the sum of millions of transactions involving individual buyers (importers) and sellers (exporters)
- Historically, international trade has been studied from the perspectives of the *countries* involved in bilateral trade flows
- In the 2000s, the use of *micro-level* data has allowed to dig into firms' trade participation
  - From the perspective of exporters deciding to serve foreign countries (Melitz, 2003)
  - From the perspective of importers deciding upon the sourcing of their inputs (Antras et al., 2014)
- Recently, a number of countries have released data at the **firm-to-firm** level

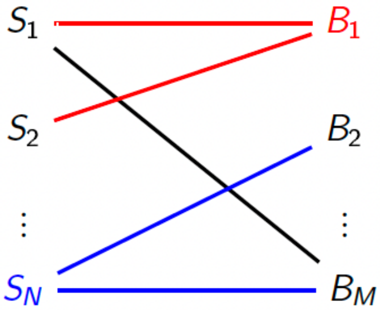
Firm-to-firm trade

Figure 1: Firm-to-Firm Trade. U.S. importers and Norwegian exporters, HS 847990, 2006.



Source: Bernard & Moxnes (2018) The picture shows all buyer-seller relationships between Norwegian exporters and US importers on a particular type of machines. Each node is a firm, and the arrows show the direction of trade

Firm-to-firm trade model



- Bipartite graph structure
- Sellers / exporters and buyers / importers are the nodes
- (Observed) transactions are the edges

New data, new questions

- **Static structure** of firm-to-firm trade networks
  - How much (more) heterogeneity?
  - Sorting? Are high-productivity exporters matched with high-productivity importers?
  - Efficiency? Do international markets help firms identify high-productivity / high-capability suppliers worldwide?
  - Market power: Pricing of inputs in firm-to-firm trade
- **Dynamics** of firm-to-firm trade relationships
  - (Intensive and Extensive) Adjustment of F2F trade relationships to shocks?
  - Pass-through of shocks from upstream to downstream firms?

New data, new challenges

- Firm-to-firm trade data can be seen as segments of **Global Value Chains**
- A substantial improvement over the literature on GVCs which mostly exploits sectoral data on trade in value added
- Still far from perfect:
  - Cannot reconstitute the whole value chain
  - Do not observe the universe of firms competing with exporters in the data

Part II

Stylized facts

The buyer extensive margin

- Buyer margin explains a large fraction of the variation in aggregate trade:

$$\ln x_j = \ln \#Exporters_j + \ln \#Products_j + \ln \#Importers_j + \ln Density_j + \ln \bar{x}_j$$

Table 2: The Margins of Trade.

VARIABLES	(1) Sellers	(2) Products	(3) Buyers	(4) Density	(5) Intensive
Exports (log)	0.57^a	0.53^a	0.61^a	-1.05^a	0.32^a
	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)
N	205	205	205	205	205
R^2	0.86	0.85	0.81	0.81	0.50

Note: We decompose total exports to country j, x_j, into the product of the number of trading firms, f, the number of traded products, p, the number of buyers, b, the density of trade, d, i.e. the fraction of all possible firm-product-buyer combinations for country j for which trade is positive, and the average value of exports, $\bar{x}$. Hence, $x_j = f_j p_j b_j d_j \bar{x}_j$, where $d_j = o_j/(f_j p_j b_j)$, o_j is the number of firm-product-buyer observations for which trade with country j is positive and $\bar{x}_j = x_j/o_j$ is average exports per firm-product-buyer. We regress the logarithm of each component on the logarithm of total exports to a given market in 2006, $\ln f_j$ against $\ln x_j$. Robust standard errors in parentheses. ^{a} p$<$ 0.01, ^{b} p$<$ 0.05, ^{c} p$<$ 0.1.

Source: Bernard, Moxnes, Ulltveit-Moe (2018). Data are for Norway in 2006

Buyers margin and gravity variables

- A firm's number of buyers is higher in larger markets and smaller in remote markets

Table 3: Within-Firm Gravity.

VARIABLES	(1) Exports	(2) # Buyers	(3) Exports/Buyer
Distance	-0.48[a]	-0.31[a]	-0.17[a]
GDP	0.23[a]	0.13[a]	0.10[a]
Firm FE	Yes	Yes	Yes
N	53,269	53,269	53,269
R^2	0.06	0.15	0.01

Note: 2006 data. Robust standard errors in parentheses, clustered by firm. [a] p< 0.01, [b] p< 0.05, [c] p< 0.1. All variables in logs.

Source: Bernard, Moxnes, Ulltveit-Moe (2018). Data are for Norway in 2006

Buyers margin and gravity variables

- A firm's number of buyers is higher in larger markets and smaller in remote markets

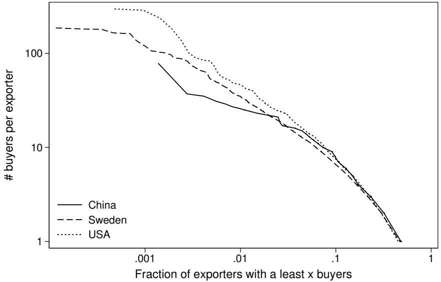
	Dependent Variable (all in log)						
	Product-level				Firm-level		
	Value of Exports	# Sellers	# Buyers per Seller	Mean export per Buyer-seller	Value of Exports	# Buyers	Exports per Buyer
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log Distance	-1.230***	-.557***	-.309***	-.364***	-.513***	-.339***	-.175***
	(.067)	(.034)	(.022)	(.045)	(.050)	(.030)	(.038)
log Import Demand	.805***	.236***	.105***	.464***	.444***	.133***	.311***
	(.016)	(.007)	(.005)	(.010)	(.014)	(.007)	(.011)
log GDP per Capita	.106***	.044**	.107***	-.046*	.025	.022	.002
	(.041)	(.018)	(.012)	(.027)	(.029)	(.015)	(.021)
Observations	67,270	67,270	67,270	67,270	481,833	481,833	481,833
R-squared	.601	.733	.373	.542	.640	.318	.655
Fixed effects	Product	Product	Product	Product	Firm × Product	Firm × Product	Firm × Product
# Fixed effects	4,374	4,374	4,374	4,374	78,609	78,609	78,609
Within R-squared	.390	.459	.256	.182	.138	.129	.068

Notes: Standard errors clustered at the country×hs2 level in parentheses with ***, ** and * respectively denoting significance at the 1, 5 and 10% levels. Data are for France in 2007. Source: Lenoir, Martin, Mejean (2018)

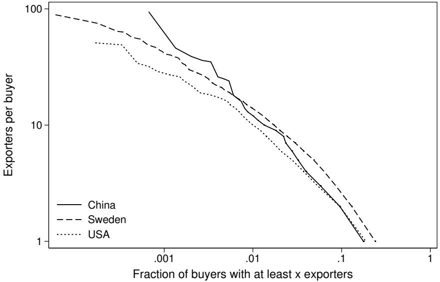
Many-to-Many / Many-to-One

- The distributions of buyers per exporter and exporters per buyer are characterized by many firms with few connections and a few firms with many connections (many-to-many matching)

buyers per exporter



exporters per buyer

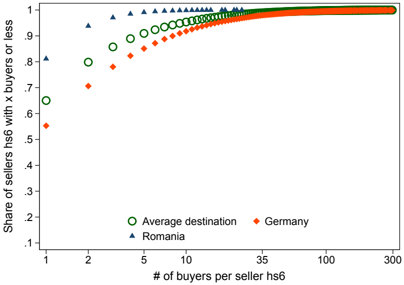


Notes: Norwegian data for 2006. The estimated slope coefficients are bw -1.02 and -1.13 for number of buyers per exporter and bw -.8 and -.92 for number of exporters per buyer. Source: Bernard et al (2018)

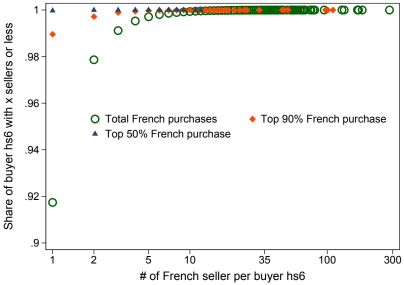
Many-to-Many / Many-to-One

- Once controlling for the product dimension, most buyers interact with a single exporter (many-to-one matching)

buyers per exporter



exporters per buyer

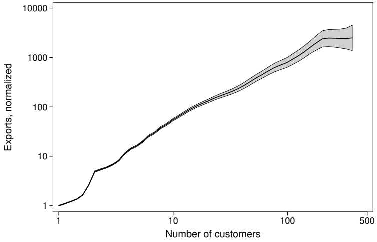


Notes: French data for 2007. Source: Lenoir et al (2018)

Buyer Margin and Seller Size

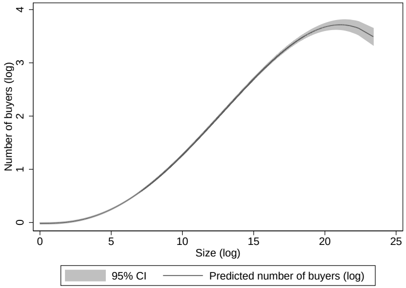
- Within a market, exporters with more customers have higher total sales, but the distribution of exports across customers does not vary systematically with the number of customers

Norwegian data



Note: 2006 data. The Figure shows the fitted line from a kernel-weighted local polynomial regression of log firm-destination exports on log firm-destination number of customers. Axes scales are in logs. Exports are normalized, see footnote 4.

French data

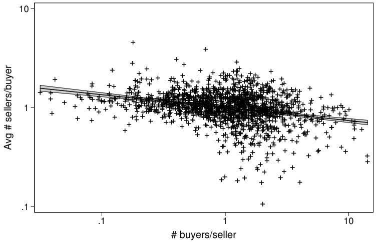


Source: Bernard et al (2018) and Lenoir et al (2018)

Assortative matching?

- Negative degree assortivity among sellers and buyers
  Note: True in terms of the firms' degree but not necessarily true in terms of the firms' total sales/purchases

Figure 6: Matching buyers and sellers across markets.



Note: 2006 data. The Figure shows all possible values of the number of buyers per Norwegian firm in a given market j, a_j, on the x-axis, and the average number of Norwegian connections among these buyers, $b_j(a_j)$, on the y-axis. Axes scales are in logs. Both variables are demeaned, i.e. we show $b_j(a_j)/\bar{b}_j(a_j)$, where $\bar{b}_j(a_j)$ is the average number of Norwegian connections among all buyers in market j. The fitted regression line and 95% confidence intervals are denoted by the solid line and gray area. The slope coefficient is -0.13 (s.e. 0.01).

Source: Bernard, Moxnes, Ulltveit-Moe (2018). Data are for Norway in 2006

Taking stock: Bernard and Zi (2023)

- The cross-sectional stylized facts that I just reviewed can be reproduced in a simple random allocation model characterized by
  1. A discrete number of sellers and buyers...
  2. of heterogeneous productivity / size ...
  3. randomly matched in product markets
- In the next lecture, I will develop such a model and discuss how the panel dimension of the data can be used to estimate the parameters of the model

This course

- Matching in frictional good markets
  1. The Ricardian economy as a limit case
  2. Introducing random search
  3. Pricing in frictional markets
- Shocks and the dynamics of trade in frictional markets
  1. Propagation of shocks in value chains
  2. Macroeconomic shocks in frictional markets

Part III

A refresher on comparative advantages

A primer on comparative advantage

- Basic question in international trade: Why do countries trade?
- Ricardo's answer: Because countries are different
  1. They have different "abilities" to produce different goods, different endowments in resources
  2. These differences generate heterogeneous relative prices in autarky...
  3. ... That are at the root of *specialization* in an open economy
- When asked to name one proposition in the social sciences that is both true and non-trivial, Paul Samuelson famously replied: "Ricardo's theory of comparative advantage"
- How true?
  1. Ricardian theory is difficult to extend to more than two countries
  2. Ricardian theory does not explain the *gravity* structure of trade
- Several attempts (Jones, 1961, Wilson, 1980) with mitigated success, until Eaton and Kortum (2002) methodological advance.

Assumptions

- I countries ($i = 1...I$)
- A continuum of goods $j \in [0,1]$
- Aggregate consumption in country i:

$$U_i = \left[\int_0^1 Q_i(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} \quad \Rightarrow \quad Q_i(j) = \left(\frac{P_i(j)}{P_i}\right)^{-\sigma} \frac{R_i}{P_i}$$

▸ demand

- Goods produced with a bundle of inputs which price is homogenous within countries c_i (first taken as exogenous)
- Iceberg trade costs $d_{ni} > 1$. Without loss of generality $d_{ii} = 1$. Cross-border arbitrage implies: $d_{ni} \leq d_{nk} d_{ki}$ ▸ Iceberg

Assumptions (ii)

- Country i's efficiency in producing good j: $z_i(j)$

⇒ (Minimum) CIF price of good j produced in country i, when exported in country n:

$$p_{ni}(j) = \underbrace{\frac{c_i}{z_i(j)}}_{\text{Unit cost}} \underbrace{d_{ni}}_{\text{Trade barrier}}$$

optimal price

- **Perfect competition** across suppliers

⇒ Price actually paid in country n for good j:

$$p_n(j) = min\{p_{ni}(j); i = 1...I\}$$

(Note that most results continue to hold with Bertrand competition)

Assumptions (iii)

- Probabilistic representation of technologies: $z_i(j)$ is the realization of a random variable Z_i drawn from a country-specific probability distribution:

$$F_i(z) = Pr[Z_i \leq z]$$

- Productivity draws assumed independent across goods and countries
- F_i assumed to be Fréchet (Type II extreme value): Fréchet

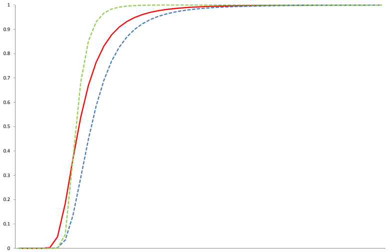
$$F_i(z) = e^{-T_i z^{-\theta}}$$

with $T_i > 0$ and $\theta > 0$

Note: Fréchet can be shown to be the outcome of a process of innovation and diffusion in which T_i is a stock of ideas. See Eaton & Kortum (IER, 1999)

Interpretation

$$F_i(z) = e^{-T_i z^{-\theta}}$$



Benchmark in red. Doubling of T_i in blue / of θ in green

- T_i "state of technology" or **absolute advantage**: Bigger T_i means that country i is more likely to draw a high efficiency for any good j
- θ heterogeneity across goods or extent of **comparative advantages** within the continuum: Bigger θ implies less variability in productivity

Price distribution

- Country i's distribution of prices in country n:

$$G_{ni}(p) \equiv Pr[P_{ni} \leq p] = 1 - e^{-T_i(c_i d_{ni})^{-\theta} p^{\theta}}$$

- Country n's actual distribution of prices:

$$\begin{aligned} G_n(p) &\equiv Pr[P_n \leq p] \\ &= 1 - \prod_i [1 - G_{ni}(p)] \\ &= 1 - e^{-\Phi_n p^{\theta}} \end{aligned}$$

where $\Phi_n \equiv \sum_i T_i(c_i d_{ni})^{-\theta}$

▸ details

Price distribution

$$G_n(p) = 1 - e^{-\Phi_n p^\theta}, \quad \Phi_n \equiv \sum_i T_i (c_i d_{ni})^{-\theta}$$

Distribution of prices governed by

- States of technology around the world $\{T_i\}$,
- Input costs around the world $\{c_i\}$,
- Geographic barriers $\{d_{ni}\}$
  - If $d_{ni} = 1, \forall n, i$ then $\Phi_n = \Phi, \forall n$ (LOP)
  - If $d_{ni} \to \infty, \forall i$ then $\Phi_n = T_n c_n^{-\theta}$ (Autarky)

$\Rightarrow$ Φ_n interprets as the strength of competition that any firm will encounter in country n

Bilateral trade

- Share of goods that n buys from i = Probability that i provides the lowest price good in country n ‣ details ‣ Link with het. consumers models

$$\begin{aligned}\pi_{ni} &= \frac{X_{ni}}{X_n} \\ &= Pr[p_{ni}(j) \leq min\{p_{ns}(j); s \neq i\}] \\ &= \int_0^\infty \prod_{s \neq i} [1 - G_{ns}(p)] dG_{ni}(p) \\ &= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}\end{aligned}$$

- or in log:

$$\ln X_{ni} = \underbrace{\ln\left(T_i c_i^{-\theta}\right)}_{Exporter\ capability} + \underbrace{\ln\left(X_n \Phi_n^{-1}\right)}_{Importer\ Market\ Potential} - \underbrace{\theta \ln d_{ni}}_{Gravity}$$

$\Rightarrow$ **Gravity-type equation**

Bilateral trade (ii)

Interpretation of the gravity equation:

- The coefficient on trade barriers relates to the distribution of productivities

⇒ The more heterogenous productivities across producers of a commodity (the lower θ), the strongest the cost advantage of the lowest cost supplier, the more likely she remains the lowest cost supplier when trade costs increase

⇒ Trade flows respond to geographic barriers at the **extensive margin**: As a source becomes more expensive or remote, it exports a narrower range of goods

General equilibrium solution

Gains from trade

- Share of domestic goods in consumption:

$$\pi_{nn} = \frac{T_n w_n^{-\theta}}{\Phi_n}$$

- Price index

$$P_n = \gamma \Phi_n^{-1/\theta}$$

- Hence real wages:

$$\frac{w_n}{P_n} = \gamma^{-1} T_n^{1/\theta} \pi_{nn}^{-1/\theta}$$

Gains from trade, cont.

- Gains from trade = Change in real wages across static equilibria:

$$GT_n \equiv \frac{w_n/P_n}{w_n'/P_n'} = \left(\frac{\pi_{nn}'}{\pi_{nn}}\right)^{1/\theta}$$

- Now you understand why EK has become the central piece in quantitative trade: Any policy experiment generates welfare consequences that can be assessed by thinking about its impact on π_{nn}, conditional on θ

Extensions

- Multiple sectors (Costinot, Donaldson & Komunjer, 2012)
- Input-output linkages (Caliendo & Parro, 2014 using the roundabout production in Krugman and Venables, 1995)
- Imperfect competition (Bernard, Eaton, Jensen & Kortum)
- Matching frictions (Lenoir, Martin & Mejean, 2020)

Part IV

Conclusion

Concluding remarks

- An elegant way of introducing Ricardo into a multi-country (eventually multi-sector) model
- Predictions consistent with the **gravity** equation
- Well-suited to GE analysis, eg on the welfare impact of trade
- Not well-suited to dig into micro-level determinants of trade
  - A maximum of one technology / firm per country serves a given market
  - Ex-post degenerated distribution of firms / technologies
- Tomorrow: Introduce frictions and recover interesting micro-level predictions

Counterfactuals

Once estimated, the model can be used to run counterfactuals:

- What are the welfare gains from trade? (Arkolakis et al, 2012)
- What is the impact of multilateral/unilateral tariff eliminations? (Caliendo & Parro, 2015)
- How much does trade spread the benefit of local improvements in technology? (Levchenko & Zhang, 2011)
- How will climate change affect the patterns of production and trade? (CDS, 2016)
- How does specialization affect the volatility of GDPs? (Caselli et al, 2015)

Other applications

- The EK framework is quite generic
- Its discrete choice fundamentals have been used for
    1. Optimal sourcing strategy of inputs by multinational firms (Tintelnot QJE15, Antras, Fort Tintelnot AER17)
    2. Mergers and Acquisitions (Head and Ries 2008)
    3. Gravity in commercial services (Head, Mayer and Ries 2009)
    4. Gravity in tourism (Faber and Gaubert AER19)
    5. Gravity in migration (many following Redding JIE2016, Redding 2020 is a good survey)
    6. Production within teams (Freund, JMP2024)
    7. ...

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Demand functions

- Consumers solves:

$$\begin{cases} \max_{\{Q_i(j)\}_{j\in[0,1]}} \left[\int_0^1 Q_i(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} \\ s.t. \int_0^1 P_i(j)Q_i(j)dj \leq R_i \end{cases}$$

- Solution of the maximization program is:

$$Q_i(j) = \left(\frac{P_i(j)}{P_i}\right)^{-\sigma} \frac{R_i}{P_i}$$

with P_i the ideal price index $(R_i/P_i = U_i, \quad \forall R_i)$:

$$P_i = \left[\int_0^1 P_i(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$$

Back to assumptions

Iceberg trade costs

Alan Deardoff's glossary: *A cost of transporting a good that uses up some fraction of the good itself, rather than other resources. By analogy with floating icebergs, costless except for the part of the iceberg that melts. Far from realistic, but a tractable way of modeling transport costs since it impacts no other market. Due to Samuelson (1954).*

- Originates in von Thunen (1826) and Samuelson (1954)
- A short-cut: Trade cost is just a deadweight loss to the economy. Avoids modelling the transport sector
- Anderson & van Wincoop (2004): A survey on the measurement of trade costs. The ad valorem equivalent is about 170% in rich countries ($d_{ni} = 2.7$). This includes transport, border-related and local distribution costs

Back to assumptions

Optimal Prices

- Firms' profit:

$$\pi_i(j) = \sum_n \left[p_{ni}(j) Q_{ni}(j) - \frac{c_i}{z_i(j)} d_{ni} Q_{ni}(j) \right] = \sum_n \pi_{ni}(j)$$

- Under perfect competition:

$$p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni}$$

and

$$Q_{in}(j) = 0 \; if \; p_{in}(j) > p_n(j) / Q_n(j) \; otherwise$$

Back to assumptions

Price distribution

- $p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni}$ is the realization of a random variable P_{ni} which cdf is:

$$\begin{aligned} G_{ni}(p) &= Pr[P_{ni} \leq p] = Pr\left[Z_i \geq \frac{c_i d_{ni}}{p}\right] \\ &= 1 - F_i\left(\frac{c_i d_{ni}}{p}\right) = 1 - e^{-T_i\left(\frac{c_i d_{ni}}{p}\right)^{-\theta}} \end{aligned}$$

- $p_n(j) = min\{p_{ni}(j); i = 1...I\}$ is the realization of a random variable $P_n = min\{P_{ni}; i = 1...I\}$ which cdf is:

$$G_n(p) = Pr[P_n \leq p] = 1 - \prod_{i=1}^{I} Pr[P_{ni} > p]$$

$$= 1 - \prod_{i=1}^{I} [1 - G_{ni}(p)] = 1 - e^{-p^{\theta} \sum_{i=1}^{I} T_i (c_i d_{ni})^{-\theta}}$$

◂ Back to the model

Price index

- Using:

$$G_n(p) = 1 - e^{-\Phi_n p^\theta} \quad and \quad g_n(p) = \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta}$$

- one can derive the price index:

$$\begin{aligned}
P_n &= \left[\int_0^1 p_n(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}} = \left[\int_0^1 p^{1-\sigma} dG_n(p)\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^1 p^{1-\sigma} \theta p^{\theta-1} \Phi_n e^{-\Phi_n p^\theta} dp\right]^{\frac{1}{1-\sigma}} \\
&= \Phi_n^{-1/\theta} \left[\int_0^1 u^{\frac{1-\sigma}{\theta}} e^{-u} du\right]^{\frac{1}{1-\sigma}} \quad where \quad u = \Phi_n p^\theta \\
&= \Phi_n^{-1/\theta} \left[\Gamma\left(\frac{1-\sigma}{\theta} - 1\right)\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

(well-defined iif $\sigma - 1 < \theta$)

Back to the model

Fréchet distribution

- Generalized extreme value distribution: A family of continuous probability distributions usually used as an approximation to model the maxima of long (finite) sequences of random variables
- CDF:

$$F(x;\mu,\sigma,\xi) = exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1\xi}\right\}$$

μ a location parameter, $\sigma > 0$ the scale parameter, ξ the shape parameter

Fréchet distribution

- In particular:
  - Gumbel or type I extreme value: $\xi = 0$

$$F(x;\mu,\sigma,0) = exp\left\{-exp\left[-\frac{x-\mu}{\sigma}\right]\right\}, \quad x \in R$$

  - Frechet or type II extreme value: $\xi = \alpha^{-1} > 0$

$$F(x;\mu,\sigma,\xi) = \begin{cases} 0, & x \leq \mu \\ exp\left\{-\left[\frac{x-\mu}{\sigma}\right]^{-\alpha}\right\}, & x > \mu \end{cases}$$

  - Reversed Weibull or type III extreme value: $\xi = -\alpha^{-1} < 0$

$$F(x;\mu,\sigma,\xi) = \begin{cases} exp\left\{-\left[-\frac{x-\mu}{\sigma}\right]^{\alpha}\right\}, & x < \mu \\ 1, & x \geq \mu \end{cases}$$

Back to the model

A model of technology diffusion (EK, IER 1999)

- A model of endogenous growth in which technology is the result of research effort
- Flow of ideas diffusing to country i $\dot{\mu}_{it}$ depends on the stock of researchers in each country, their productivity and the rate at which ideas diffuse across countries
- Quality of an idea is a random variable drawn in a Pareto $F(z) = 1 - z^{-\theta}$
- New ideas adopted at a rate $\dot{\mu}_{it} z^{-\theta}$

$\Rightarrow$ Proba that no idea is adopted in the time interval $[t, t+dt] = e^{-\dot{\mu}_{it} z^{-\theta} dt}$

Evolution of the production frontier: $H_i(z, t+dt) = H_i(z,t) e^{-\dot{\mu}_{it} z^{-\theta} dt} \Rightarrow \frac{\partial \ln H_i(z,t+dt)}{\partial t} = -\dot{\mu}_{it} z^{-\theta} \Rightarrow H_i(z,t) = e^{-\mu_{it} z^{-\theta}}$

Back to the model

Details on trade shares

- Probability that country i is the lowest-cost supplier:

$$
\begin{aligned}
\pi_{ni} &= \int_0^\infty \prod_{s \neq i} Pr[P_{ns} \geq p] dG_{ni}(p) \\
&= \int_0^\infty e^{-p^\theta \sum_{s \neq i} T_s (c_s d_{ns})^{-\theta}} dG_{ni}(p) \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \int_0^\infty \Phi_n e^{-p^\theta \Phi_n} \theta p^{\theta - 1} dp \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} [1 - G_n(p)]_0^\infty \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}
\end{aligned}
$$

Details on trade shares

- Distribution of prices conditional on being the lowest cost supplier:

$$
\begin{aligned}
\tilde{G}_{ni}(p) &= Pr[P_{ni} \leq p | P_{ni} \leq min_{s \neq i}\{P_{ns}\}] \\
&= \int_0^p \prod_{s \neq i} Pr[P_{ns} \geq q] dG_{ni}(q) \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \int_0^p \Phi_n e^{-p^{\theta}\Phi_n} \theta p^{\theta - 1} dp \\
&= \pi_{ni} G_n(p)
\end{aligned}
$$

For goods that are purchased, conditioning on the source has no bearing on the good's price $\rightarrow$ Trade shares only depends on π_{ni}

Back to the model

Link with heterogeneous consumers models

- For each good, the probability of buying from i is:

$$\begin{aligned}\mathbb{P}_{ni} &= Pr\left[p_{ni}(j) < p_{ns}(j), \quad \forall s \neq i\right] \\ &= Pr\left[\ln Z_i > \ln \frac{c_i d_{ni}}{c_s d_{ns}} + \ln Z_s, \quad \forall s \neq i\right]\end{aligned}$$

- Since Z_i is distributed Fréchet, $\ln Z_i$ is distributed Gumbel $\Rightarrow$ multinomial logit:

$$\mathbb{P}_{ni} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\sum_s T_s \left(c_s d_{ns}\right)^{-\theta}}$$

- Very similar to heterogeneous consumers models (MacFadden, Anderson et al)

Link with heterogeneous consumers models

- Suppose indirect utility of consumer u is:

$$U_i(u) = U_i - p_i + \varepsilon_i(u)$$

with $\varepsilon_i(u)$ drawn from a Gumbel distribution with CDF

$$Pr\left(\varepsilon_i(u) \leq \varepsilon\right) = exp\left(-exp(-\theta\varepsilon)\right)$$

- Logit: For each consumer, the probability of buying i is the probability that the indirect utility of buying from i is larger than the utility of buying from any other supplier:

$$\mathbb{P}_i = \frac{exp\left[\theta(U_i - p_i)\right]}{\sum_j exp\left[\theta(U_j - p_j)\right]}$$

- EK: $\ln p_i(u) = \ln c_i - \ln Z_i(u)$ and $\ln Z_i(u)$ distributed Gumbel thus the probability of buying i is the probability that $\ln p_i(u)$ is minimum:

$$\mathbb{P}_i = \frac{exp\left[\theta(-\ln c_i)\right]}{\sum_j exp\left[\theta(-\ln c_j)\right]} = \frac{c_i^{-\theta}}{\sum_j c_j^{-\theta}}$$

General equilibrium solution

- Suppose production is linear in labor:

$$c_i = w_i$$

$\Rightarrow$ Price levels as a function of wages:

$$P_n = \gamma \left[\sum_i T_i \left(d_{ni} w_i \right)^{-\theta} \right]^{-1/\theta}$$

where $\gamma \equiv \left[\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right]^{1/(1-\sigma)}$ price index

$\Rightarrow$ Trade shares as a function of wages and prices:

$$\frac{X_{ni}}{X_n} = T_i \left(\frac{\gamma d_{ni} w_i}{P_n} \right)^{-\theta}$$

General equilibrium solution

- To close the model, one needs to solve for equilibrium wages across countries
  - Output is distributed to workers:
  $$Y_i = w_i L_i = X_i$$
  - Market clearing condition
  $$Y_i \sum_n X_{ni} = \sum_n \pi_{ni} X_n$$
  - Which gives a system of $I-1$ independent equations (Walras' law) that can be solved for nominal wages (up to a numeraire):
  $$w_i L_i = \sum_n \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_s T_s (w_s d_{ns})^{-\theta}} w_n L_n$$
- Can be solved numerically as a fixed point problem