

## Lecture 3: International trade under imperfect competition

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# Introduction

- Limitation of the classics : Unable to explain trade between similar countries, gravity, intra-industry trade
- Theoretical response : **imperfect competition**

## ⇒ “New” theories of international trade

- Limitation of imperfect competition models : extensive versus intensive margins, impact of productivity on trade
- Theoretical response : imperfect competition with **heterogeneous firms**

## ⇒ “New-new” theories of international trade

# “Old” and “New” theories of international trade

## Old theories (Ricardo, HOS) :

- Perfect competition and constant returns to scale
- Homogenous goods
- PPF different across countries

## Consequences :

- Explains *inter-industry* trade between countries with *different endowments* (North-South trade)
- Does not explain *intra-industry* trade between countries with *similar endowments* (North-North trade)
- Market size irrelevant

## New theories :

- Increasing returns to scale (fixed costs) → Imperfect competition
- Products are differentiated
  - Horizontally (varieties)
  - Vertically (qualities)
- Preference for diversity

## References :

- Robinson (1930), Chamberlin (1936)
- Lancaster (1980), Helpman (1981)
- Krugman (1979, 1980) Helpman, Krugman (1985)

# Intra-Industry Trade

## Top-10 country pairs (% of bilateral trade, 2000)

Top total IIT shares (per cent)		
Germany	France	86.20
Netherlands	Belgium and Luxembourg	85.01
France	Belgium and Luxembourg	80.42
France	United King- dom	77.08
Germany	Switzerland	76.99
Germany	Belgium and Luxembourg	76.83
Austria	Germany	76.63
France	Spain	76.55
Germany	Netherlands	76.01
Canada	United States	73.55

Source : Fontagné, Freudenberg & Gaulier (2006)

# Class Overview

1. The Krugman model
2. Specialization in the Helpman-Krugman model
3. Heterogeneous firms in the Melitz model

# The Krugman (1980) Model

# Ingredients

- Increasing returns to scale (fixed cost of producing)
- Monopolistic competition
- Iso-elastic preferences
- Proportional transportation costs

## Consumption (Dixit-Stiglitz)

- L identical individuals who work, consume and hold the firms. Equivalent to one representative household supplying L units of labor.
- The representative agent consumes a continuum of **differentiated goods**  $\omega \in \Omega$  :

$$\max_{q(\omega)} U = \max_{q(\omega)} \left( \int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

with  $\sigma > 1$  the **(constant) elasticity of substitution** between varieties (number of varieties irrelevant)

- Under the budget constraint :  $\int_{\Omega} p(\omega)q(\omega)d\omega = wL$  (no capital, no profit in equilibrium)
- This yields the **demand function** :

$$q(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P} \text{ with } P = \left( \int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

# Interpretation of $P$

- $P$  is the **consumer price index** (also called “ideal price index”), aggregate of  $p(\omega)$  such that the utility of the real income (i.e. nominal income divided by the price index) is the same whatever the general level of prices.
- With  $P$  calculated along the above formula, the utility of  $wL/P$  is independent of the general price level. Demonstration : Compute  $U$  based on optimal  $q(\omega)$  :

$$\begin{aligned}
 U &= \left( \int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\
 &= \left( \int_{\Omega} \left( \frac{p(\omega)}{P} \right)^{-\sigma \frac{\sigma-1}{\sigma}} \left( \frac{wL}{P} \right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\
 &= \frac{wL}{P} P^{\sigma} \left( \int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\
 &= \frac{wL}{P}
 \end{aligned}$$

- Aggregate utility  $U$  is equal to the real income : if both the nominal income  $wL$  and the price index  $P$  increase by x%, utility stays unchanged.

## Interpretation of $P$ (2)

- At the consumer's optimum, we have  $P = \left( \int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$  and  $U = \frac{wL}{P}$
- Since  $\sigma > 1$ , the price index  $P$  is lower than the simple average of prices  $p(\omega)$  :  
$$P = \left( \int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} < \int_{\Omega} p(\omega) d\omega$$
- This comes from consumer's preference for diversity : higher diversity increases utility, for given prices  $p(\omega)$  ; it is as if real income was growing thanks to a lower price index  $P$
- For a given nominal income  $wL$ , the price index  $P$  varies inversely to utility
- **In the Krugman (1980) model, international trade raises utility through a rise in the diversity of products available to the consumer (not through efficiency gains as in the Ricardian and HOS models of trade).**

# Production

- Each firm produces one variety  $\omega$  for which it has a **monopole** (preference for diversity+no additional cost of creating a variety  $\rightarrow$  No incentive to compete on an existing variety)
  - **Fixed cost** : to produce  $q(\omega)$ , the firm uses a volume of labor equal to :  

$$l(q(\omega)) = f + \frac{q(\omega)}{\varphi}$$
  - **Optimal price** :  $p = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$  with  $\frac{\sigma}{\sigma-1}$  the markup
  - **Profit** :  $\pi(\omega) \equiv p(\omega)q(\omega) - w \left( f + \frac{q(\omega)}{\varphi} \right) = w \left( \frac{q(\omega)}{(\sigma-1)\varphi} - f \right)$
  - **Free entry** :  $\pi(\omega) = 0 \Rightarrow q(\omega) = (\sigma - 1)\varphi f$
- $\Rightarrow$  All firms produce the same quantity at the same price ( $w$  omitted in the following)
- **Number of firms** :  $n$  such that  $n \left( f + \frac{q}{\varphi} \right) = L \Rightarrow n = \frac{L}{\sigma f}$
- $\Rightarrow$  The number of firms depends on the size of the country ( $L$ ), on fixed costs ( $f$ ) and on the elasticity of substitution  $\sigma$ . Higher fixed costs or more competition across varieties (higher  $\sigma$ ) will reduce the number of firms in the long run. This is because both  $\sigma$  and  $f$  reduce the profit of each firm

## Back to the price index

- Price of each variety :  $p = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$

⇒ Equilibrium price index :

$$P = \left( \int_{\Omega} \left( \frac{\sigma}{\sigma-1} \frac{w}{\varphi} \right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1} \frac{w}{\varphi} \left( \int_{\Omega} d\omega \right)^{\frac{1}{1-\sigma}}$$

$$\Leftrightarrow P = \frac{\sigma}{\sigma-1} \frac{w}{\varphi} n^{\frac{1}{1-\sigma}}$$

ie increasing the number of varieties reduces the price index

- Replace  $n$  by its equilibrium value :

$$P = \frac{\sigma}{\sigma-1} \frac{w}{\varphi} \left( \frac{L}{\sigma f} \right)^{\frac{1}{1-\sigma}}$$

⇒ **The price index is lower the larger the economy (high  $L$ ) because this allows more varieties to co-exist. Since utility is inversely related to the consumer price index, welfare is higher in a larger economy (in autarky).**

## Two countries

- Assume there are two identical countries except for their size :  $L, L^*$ .
- Transportation costs are of the iceberg type : when 1 unit is shipped by the exporter, the importer only receives  $1/\tau$  units, with  $\tau > 1$ . The rest has melted away.
- Prices : Domestic market :  $p^D = \frac{\sigma}{\sigma-1} \frac{w}{\varphi} \equiv p$  / Foreign market :  
 $p^X = \tau \frac{\sigma}{\sigma-1} \frac{w}{\varphi} = \tau p$

Note that the price before transportation (FOB price) is the same on both markets because the elasticity of substitution is the same and is constant. The price in the destination market (CIF price) is multiplied by  $\tau$ , ie the transportation cost is fully passed on the consumer

## Two countries (2)

- Total production :  $q = q^D + \tau q^X$
  - Total profit :  $\pi = pq - w \left( f + \frac{q}{\varphi} \right) = \frac{w}{(\sigma-1)\varphi} q - wf$
  - Free entry :  $\pi = 0 \Rightarrow q = (\sigma - 1)\varphi f$
  - Number of firms :  $n$  such that  $n \left( f + \frac{q}{\varphi} \right) = L \Rightarrow n = \frac{L}{\sigma f}$
- ⇒ **The number of firms and individual production levels are the same as in the autarky case** because (1) labor is immobile; (2) the fixed cost and the elasticity of substitution have remained unchanged. ( $\neq$  Krugman (1979) with non-CES preferences, where opening up the economy reduces  $n$ )

# International Trade

- Value of aggregate exports :  $X = n\tau p q^X(\tau p)$

with :

$$q^X(\tau p) = \left(\frac{\tau p}{P^*}\right)^{-\sigma} \frac{w^* L^*}{P^*}$$

$$p = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}$$

$$n = \frac{L}{\sigma f}$$

$$\Rightarrow X = \frac{1}{\sigma f} \left(\frac{\sigma}{(\sigma - 1)\varphi}\right)^{1-\sigma} LL^* \left(\frac{\tau W}{P^*}\right)^{1-\sigma} w^*$$

or in log :

$$\ln X = -\ln(\sigma f) + (1-\sigma) \ln \frac{\sigma}{(\sigma - 1)\varphi} + \ln L + \ln L^* + (1-\sigma) \ln \frac{\tau W}{P^*} + \ln w^*$$

# International Trade (2)

- **Gravity model :**

- Trade between two countries depends on the product of their sizes ( $LL^*$ ) and on bilateral transportation costs ( $\tau$ , usually proxied with geographic distance and other dummies)

- **Intensive and extensive margins :**

- If  $\tau \rightarrow \infty$ , then no trade
- When  $\tau$  is no longer infinite, each country starts exporting all its varieties and to import all the other country's varieties : there is a sudden rise in trade through extensive margins
- Then, while  $\tau$  continues to fall, exports of each variety increase but the number of exported varieties stays constant : trade grows through intensive margins.

# The gravity equation

	(1)	(2)	(3)	(4)	(5)	(6)
ln Pop, $i$	0.978 <sup>a</sup> (0.006)	0.893 <sup>a</sup> (0.009)	0.290 <sup>a</sup> (0.046)			
ln Pop, $j$	0.837 <sup>a</sup> (0.006)	0.835 <sup>a</sup> (0.008)	0.962 <sup>a</sup> (0.040)			
ln GDP/Pop, $i$	1.118 <sup>a</sup> (0.007)	0.921 <sup>a</sup> (0.010)	0.732 <sup>a</sup> (0.015)			
ln GDP/Pop, $j$	0.945 <sup>a</sup> (0.007)	0.702 <sup>a</sup> (0.010)	0.634 <sup>a</sup> (0.015)			
ln Dist (avg)	-1.035 <sup>a</sup> (0.014)	-1.197 <sup>a</sup> (0.015)				
Shared Language	0.506 <sup>a</sup> (0.034)	0.522 <sup>a</sup> (0.038)				
Shared Legal Origins	0.313 <sup>a</sup> (0.026)	0.160 <sup>a</sup> (0.029)				
Colonial History	1.560 <sup>a</sup> (0.380)	2.605 <sup>a</sup> (0.206)				
RTA	0.958 <sup>a</sup> (0.044)	0.593 <sup>a</sup> (0.026)	0.521 <sup>a</sup> (0.027)	0.400 <sup>a</sup> (0.029)	0.411 <sup>a</sup> (0.034)	0.317 <sup>a</sup> (0.033)
Both GATT	0.125 <sup>a</sup> (0.020)	0.155 <sup>a</sup> (0.016)	0.159 <sup>a</sup> (0.017)	0.244 <sup>a</sup> (0.038)	0.368 <sup>a</sup> (0.041)	0.206 <sup>a</sup> (0.042)
Currency union	0.688 <sup>a</sup> (0.091)	0.483 <sup>a</sup> (0.064)	0.486 <sup>a</sup> (0.068)	0.499 <sup>a</sup> (0.047)	0.469 <sup>a</sup> (0.056)	0.309 <sup>a</sup> (0.089)
Tetrads:				GBR,FRA	USA,DEU	CHE,CAN
Fixed Effects:	None	Dyads(RE)	Dyads	Tetrads	Tetrads	Tetrads
# Obs.	618233	618233	618233	665531	651603	633190
RMSE	2.165	1.480	1.473	1.677	1.722	1.832

Source : Head, Mayer and Ries (2008)

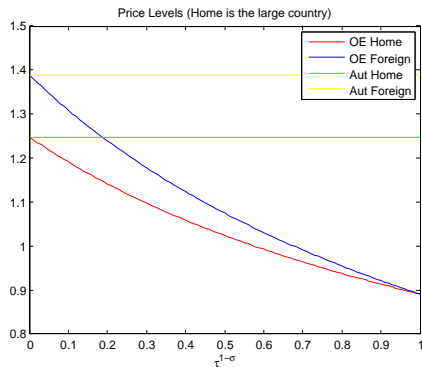
## Welfare gains

- Autarky :  $P = pn^{\frac{1}{1-\sigma}}$  and  $P^* = p^* n^{*\frac{1}{1-\sigma}}$
  - Open economies :  $P = [p^{1-\sigma} n + (\tau p^*)^{1-\sigma} n^*]^{\frac{1}{1-\sigma}}$  and  $P^* = [p^{*1-\sigma} n^* + (\tau p)^{1-\sigma} n]^{\frac{1}{1-\sigma}}$
  - Without transportation costs :  

$$P = P^* = (2np^{1-\sigma})^{\frac{1}{1-\sigma}} < (np^{1-\sigma})^{\frac{1}{1-\sigma}} \text{ since } \sigma > 1$$
- ⇒ Opening up the economy yields a welfare gain deriving from more diversity. In Krugman (1979), there is also a pro-competitive effect (fall in  $p$  due to a rise in  $\sigma$ )**

# Welfare Gains (2)

## Prices as a function of the “freeness” of trade



# Wages

Trade Balance :

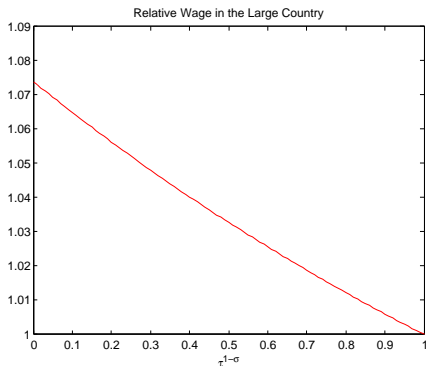
$$\underbrace{\lambda \times L \times L^* \times \left(\frac{\tau W}{P^*}\right)^{1-\sigma} \times w^*}_X = \underbrace{\lambda \times L \times L^* \times \left(\frac{\tau W^*}{P}\right)^{1-\sigma} \times w}_{X^*}$$

$$\Rightarrow \frac{w}{w^*} = \left( \frac{LW^{1-\sigma} + L^*(\tau W^*)^{1-\sigma}}{L(\tau W)^{1-\sigma} + L^*W^{*1-\sigma}} \right)^{1/\sigma}$$

- ⇒ Without transport costs ( $\tau = 1$ ), wages are equalized across countries
- ⇒ With high transport costs ( $\tau \rightarrow \infty$ ) :  $\frac{w}{w^*} \rightarrow \left(\frac{L}{L^*}\right)^{\frac{1}{2\sigma-1}}$ , ie wages are higher in the larger country

# Wages (2)

Relative wage in the large country, as a function of the “freeness” of trade



# Interpretation

- **Absent transportation costs**, consumers in both countries have access to all varieties in the same conditions. Prices equalize across countries and trade is balanced.
- With a transportation cost, **prices are lower in the larger country** (if  $L > L^*$ ,  $P < P^*$ ) → Demand for imports is lower (it rises with  $P$ ). The big country already has access to more varieties than the small one.
- In order for trade to be balanced, exports of the big country should be lowered through a **higher marginal cost** :  $w > w^*$
- Consequence : if mobile, workers should agglomerate in the big country (because  $w > w^*$ ) : this is the foundation of the **new economic geography**, which also relies on monopolistic competition (see next class)

# Specialization : The Helpman-Krugman Model

# Assumptions

- **Two sectors** : one of differentiated goods (CES aggregator, cf. previous analysis) and one of homogeneous goods. Consumers spend a fixed share  $\mu$  of their budget in differentiated goods (Cobb-Douglas utility function)
- **The homogeneous good** is produced under constant returns to scale ( $Y = AL$ ) and is traded at no cost. Both countries produce this good ("no full-specialization" hypothesis). Marginal productivity is normalized to 1  $\Rightarrow$  Price and wage levels are equal to 1 in this sector for both countries. Since labor is mobile across sectors, wages are equalized across countries
- **In the differentiated good sector**, as in Krugman (1980), all firms produce the same quantity  $q$  that they sell at the same price  $p$  :

$$q = q^d + \tau q^X = \mu \left( \frac{p}{P} \right)^{-\sigma} \frac{wL}{P} + \tau \mu \left( \frac{\tau p}{P^*} \right)^{-\sigma} \frac{wL^*}{P^*}$$

# Specialization

- Since  $q = q^*$  (zero-profit equilibrium), it can be shown that :

$$n \left( 1 - \tau^{1-\sigma} \frac{L}{L^*} \right) = n^* \left( \frac{L}{L^*} - \tau^{1-\sigma} \right)$$

Which allows to calculate the **allocation of differentiated producers across countries** as a function of  $L/L^*$  and  $\tau$

- Denote  $s_n \equiv n/(n + n^*)$  the share of firms located in Home and  $s_L \equiv L/(L + L^*)$  the share of Home in the world (real) GDP, we have :

$$s_n = \frac{s_L(1 + \tau^{1-\sigma}) - \tau^{1-\sigma}}{1 - \tau^{1-\sigma}}, \quad \frac{ds_n}{ds_L} > 0$$

- Note :  $s_n \in [0, 1] \Rightarrow$  If  $s_L < \tau^{1-\sigma}/(1 + \tau^{1-\sigma})$  then  $s_n = 0$ . If  $s_L > 1/(1 + \tau^{1-\sigma})$  then  $s_n = 1$

## Specialization (2)

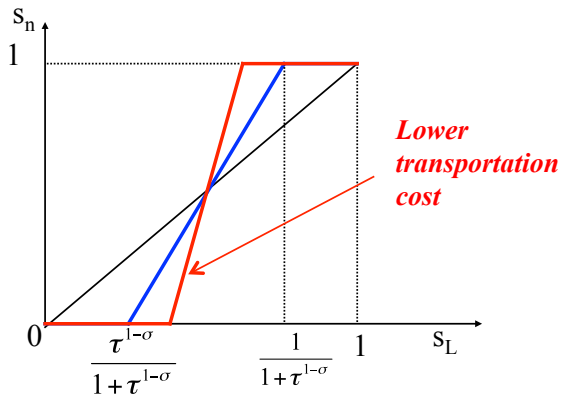
- For  $s_L < \tau^{1-\sigma}/(1 + \tau^{1-\sigma})$  or  $s_L > 1/(1 + \tau^{1-\sigma})$ , full-specialization of one country in the production of differentiated goods  $\Rightarrow$  **The size range where both countries produce the differentiated good is smaller the smaller transportation costs**
- Between the two thresholds, the larger country hosts a higher proportion of output than its share in the global population :

$$s_n = s_L + \frac{1}{2} \frac{\tau^{1-\sigma}}{1 - \tau^{1-\sigma}} (s_L - \frac{1}{2}) > s_L \quad \text{if} \quad s_L > \frac{1}{2}$$

Moreover, the share in output grows faster than the share in population ( $\frac{ds_n}{ds_L} > 1$ ). This is the **home market effect**

- A smaller transportation cost reinforces this effect

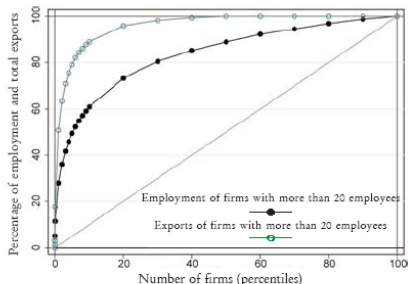
## Specialization (3)



# Limits of the model

- Exporting firms are larger and more productive than strictly domestic firms  $\Rightarrow$  different  $\varphi$ ,  $q$ ,  $p$

## Inequalities between firms, in terms of jobs and exports

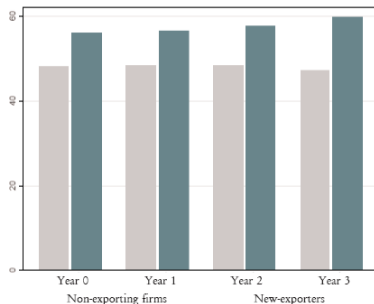


*Interpretation:* amongst the French firms with more than 20 employees, the 20% biggest exporters are responsible for 94% of total exports, but the 20% biggest employers only represent about 75% of total jobs.

*Source:* French Customs and Excise statistics and Annual Business Survey (INSEE), CEPII calculations.

Source : Crozet & Mayer (2007)

## Productivity of firms with more than 20 employees that enter the export markets



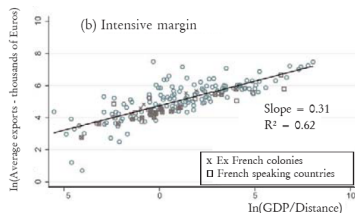
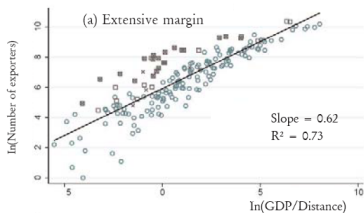
Source : French Customs and Excise statistics and Annual Business Survey (INSEE), CEPII calculations.

Source : Crozet & Mayer (2007)

## Limits of the model (2)

- Trade grows through both intensive and extensive margins (ie volume of export by firms versus number of exporting firms)

### Intensive and extensive margins of international trade, 2003



Source : Crozet & Mayer (2007)

Interpretation : A point is a destination country for French exports. Closer/Bigger countries (in terms of GDP) are served by more firms and each firm export more, on average. “Cultural” proximity increases the extensive margin.

# Heterogeneity : The Melitz (2003) Model

# Ingredients

- Monopolistic competition
- Iso-elastic preferences
- Increasing returns to scale
- Proportional transportation cost
- **Fixed cost of export**
- **Heterogeneous firms** in terms of productivity (random)

# Assumptions

## - From Krugman :

- CES preferences :

$$\left\{ \begin{array}{l} \max_{q(\omega)} \left( \int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ \text{sc} \quad \int_{\Omega} p(\omega)q(\omega) = R \end{array} \right. \Rightarrow \begin{array}{l} q(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} \frac{R}{P} \\ \text{with } P = \left( \int_{\Omega} p(\omega)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \end{array}$$

with  $R$  the nominal revenue ( $=wL +$  residual profits)

- Production Costs :  $l(q) = f + \frac{q}{\varphi} \Rightarrow$  Optimal price :  $p = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$
- Normalization of wage (cf. existence of an homogeneous good with constant returns to scale and free trade, as in Helpman-Krugman)

## Assumptions (2)

### - New :

- Labor productivity is **heterogeneous across firms**  $\Rightarrow$  Prices and quantities are now heterogeneous across firms :

$$p(\varphi) = \frac{\sigma}{\sigma-1} \frac{1}{\varphi}, \quad q(\varphi) = \left( \frac{\sigma}{(\sigma-1)\varphi} \right)^{-\sigma} \frac{R}{P^{1-\sigma}}$$

$$\Rightarrow \pi(\varphi) = \left( \frac{\sigma-1}{\sigma} \varphi P \right)^{\sigma-1} \frac{R}{\sigma} - f$$

- **Random productivities** : Productivities are drawn from a common distribution  $g(\varphi)$  which has positive support over  $[0, \infty)$  and a cdf  $G(\varphi)$  + Each period, the firm has a probability  $\delta$  of a bad shock forcing it to exit
- **Fixed (sunk) cost to enter** the market  $f^e$  (paid before discovering productivity) + **Fixed cost of exporting**  $f^X$  (on top of fixed producing cost  $f$ )

# Price Index

- Ex-post, there is a mass  $M$  of firms producing, with a productivity distribution  $\mu(\varphi)$ . With a constant death probability,  $\mu(\varphi)$  is exogenously determined by  $g(\varphi)$  and  $\delta$
- The price index writes :

$$\begin{aligned}
 P &= \left( \int_0^\infty M \mu(\varphi) p(\varphi)^{1-\sigma} d\varphi \right)^{\frac{1}{1-\varphi}} \\
 &= \left( M \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \int_0^\infty \mu(\varphi) \varphi^{\sigma-1} d\varphi \right)^{\frac{1}{1-\varphi}}
 \end{aligned}$$

- Denoting  $\tilde{\varphi} \equiv \left( \int_0^\infty \mu(\varphi) \varphi^{\sigma-1} d\varphi \right)^{\frac{1}{\sigma-1}}$  the mean productivity of firms producing (weighted average where weights are correlated to the productivity of firms), we have :  $p(\tilde{\varphi}) = \frac{\sigma}{(\sigma-1)\tilde{\varphi}}$  and
 
$$P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi})$$

⇒ The price index is reduced (ie welfare increases) with **more varieties or a higher average productivity** of firms on the market

# Autarky Equilibrium

- Firms face a twofold decision : i) Whether to pay the sunk entry cost  $f^e$  allowing to draw a productivity level, ii) once productivity is discovered, whether to produce (conditional on the productivity draw)
- ii) Once productivity is known, the firm exits if  $\pi(\varphi) < 0$ . If  $\pi(\varphi) \geq 0$ , it produces every period until being hit by the death shock
  - $\Rightarrow$  There is a **productivity cut-off**  $\bar{\varphi}^A$  below which firms do not produce
  - $\Rightarrow$  **“Zero cut-off profit condition”** :  $\pi(\bar{\varphi}^A) = 0$
  - $\Rightarrow$  **Ex-post distribution of productivities** : Since  $\delta$  is assumed exogenous to productivity, the exit process does not affect the distribution :  $\mu(\varphi)$  is the conditional distribution of  $g(\varphi)$  on  $[\bar{\varphi}^*, \infty)$  :

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\bar{\varphi}^A)} & \text{if } \varphi \geq \bar{\varphi}^A \\ 0 & \text{otherwise} \end{cases}$$

Remark :  $1 - G(\bar{\varphi}^A)$  is the ex-ante probability of a successful entry

$$\Rightarrow \tilde{\varphi}(\bar{\varphi}^A) = \left( \frac{1}{1-G(\bar{\varphi}^A)} \int_{\bar{\varphi}^A}^{\infty} g(\varphi) \varphi^{\sigma-1} d\varphi \right)^{\frac{1}{\sigma-1}}$$

## Autarky Equilibrium (2)

- i) Firms have an incentive to pay the fixed entry cost since, on expectations, future profits are positive (the cut-off firm is the only one with zero profits ex-post)  $\Rightarrow$  Present value of the average profit flows :

$$v(\tilde{\varphi}) = \sum_{t=0}^{\infty} (1 - \delta)^t \pi(\tilde{\varphi}) = \frac{\pi(\tilde{\varphi})}{\delta}$$

$\Rightarrow$  Net value of entry :

$$\frac{1 - G(\bar{\varphi}^A)}{\delta} \pi(\tilde{\varphi}) - f^E$$

In a free-entry equilibrium, the net value of entry is equal to zero (if negative, no firm enters/ if positive, more firms enter). This is **the Free-Entry condition**

- Together, the ZP and the FE conditions determine the autarky productivity cut-off and mean profit. Melitz demonstrates the existence and uniqueness of this equilibrium. Prices, mass of firms, etc. then come easily. One can show  $M$  is proportional to the size of the country.

# Exports

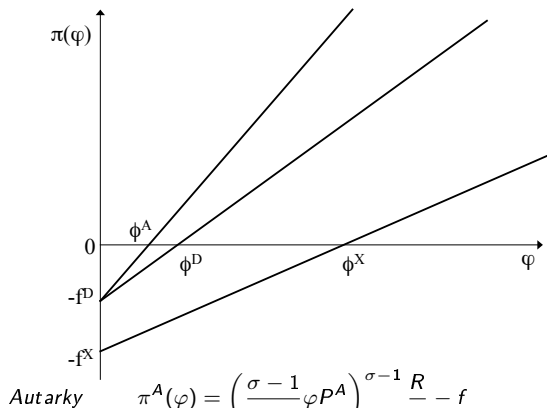
- As in Krugman, there is an iceberg trade cost  $\tau > 1$ . In addition, there is a fixed cost for exporting  $\Rightarrow$  With symmetric countries :

$$p^X(\varphi) = \frac{\tau\sigma}{(\sigma-1)\varphi}, \quad q^X(\varphi) = \left( \frac{\tau\sigma}{(\sigma-1)\varphi} \right)^{-\sigma} \frac{R}{P^{1-\sigma}}$$

$$\pi^X(\varphi) = \left( \frac{\sigma-1}{\sigma\tau} \varphi P \right)^{\sigma-1} \frac{R}{\sigma} - f^X$$

- The firm now faces another decision, whether to export, conditional on her productivity. She exports if  $\pi^X(\varphi) \geq 0 \Rightarrow$  Export productivity cut-off  $\bar{\varphi}^X$  defined by  $\pi^X(\bar{\varphi}^X) = 0$
- Moreover, the productivity cut-off to produce for the domestic market is not the same as in autarky : i) the structure of market is different, foreign firms now enter  $P$  which affects  $\pi(\varphi)$  and ii) the present value of expecting profits now integer export profits  $\Rightarrow$  Both the ZP and the FE conditions change  $\Rightarrow \bar{\varphi}^D \neq \bar{\varphi}^A$
- Note :  $\bar{\varphi}^X > \bar{\varphi}^D$  if  $\tau^{\sigma-1} f^X > F \Rightarrow$  Exporters are more productive than domestic firms, on average (Empirically ubiquitous)

## Profits

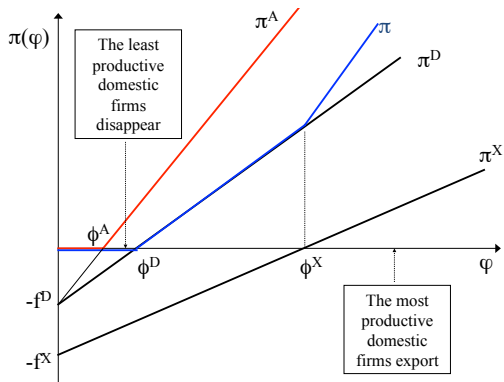


Autarky  $\pi^A(\varphi) = \left(\frac{\sigma-1}{\sigma} \varphi P^A\right)^{\sigma-1} \frac{R}{\sigma} - f$

OE, DomSales  $\pi^D(\varphi) = \left(\frac{\sigma-1}{\sigma} \varphi P^{OE}\right)^{\sigma-1} \frac{R}{\sigma} - f, \quad P^{OE} < P^A$

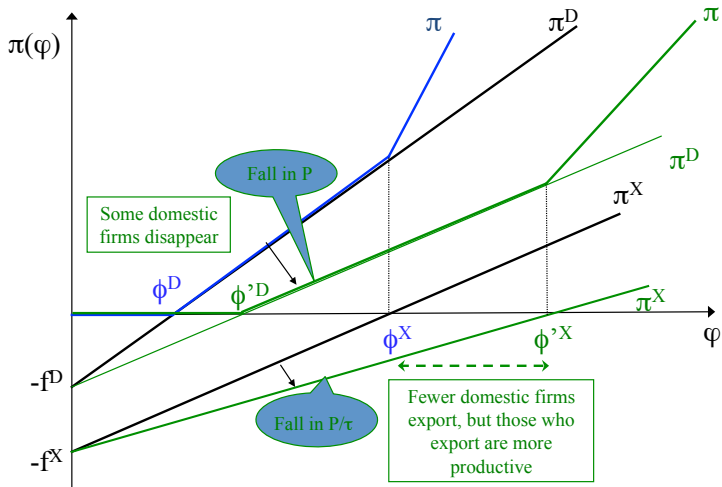
OE, XSales  $\pi^X(\varphi) = \left(\frac{\sigma-1}{\sigma\tau} \varphi P^{OE}\right)^{\sigma-1} \frac{R}{\sigma} - f^X, \quad P^{OE}/\tau < P^{OE}$

# Opening Up

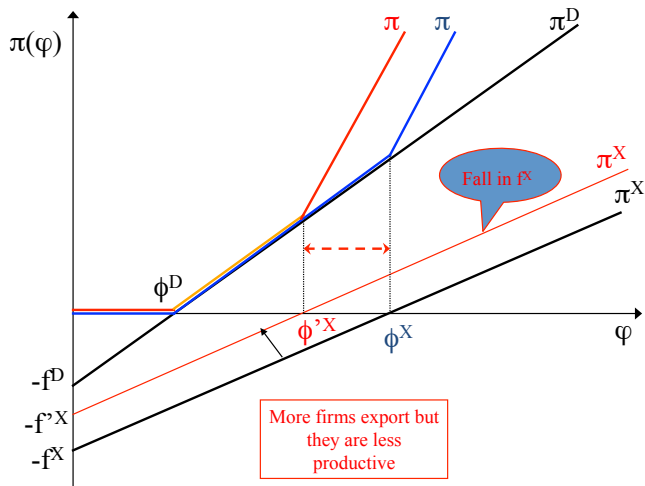


Remark : Without trade costs, openness to trade is welfare-improving due to an increase in the number of varieties available to consumers. Similar to Krugman. No reallocation effects

## Fall in the variable trade cost



## Fall in the fixed export cost



# Policy implications

- Trade policies induce **reallocations across firms** : between industries, but also within industries/between firms.
- These intra-industry reallocations are not accounted for in standard CGE-based evaluations of trade policies. Such mismatch may contribute to explain resistance to trade liberalization.
- **Two trade costs** should be distinguished :
  - + Variable costs (transportation costs, duties) : reducing these costs leads to a selection effect (less productive firms stop exporting) ; trade increases through the intensive margin (fewer firms export more)
  - + Fixed costs : information, regulations, bureaucracy, red tape : reducing these costs allows a larger number of firms, possibly less productive, to export ; trade increases through the extensive margin
- Geographic distance does not only cover transportation costs, but also cultural and regulatory distance, which are fixed costs. This may explain (i) the impact of distance on the extensive margin, and (ii) the persistent impact of distance on trade despite falling transportation costs and tariffs.

# The gravity equation with extensive margin

**Table:** Decomposition of French aggregate exports (34 industries, 159 countries, 1986-1992)

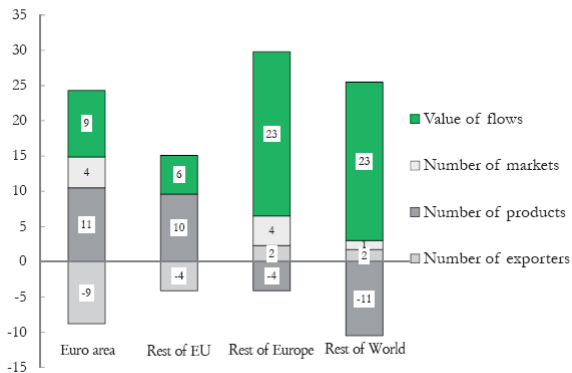
	All firms		Single-region firms	
	Average shipment	Number of shipments	Average shipment	Number of shipments
$\ln GDP_{kj}$	0.461 <sup>a</sup> (.007)	0.417 <sup>a</sup> (.007)	0.421 <sup>a</sup> (.007)	0.417 <sup>a</sup> (.008)
$\ln Dist_j$	-0.325 <sup>a</sup> (.013)	-0.446 <sup>a</sup> (.009)	-0.363 <sup>a</sup> (.012)	-0.475 <sup>a</sup> (.009)
$contig_j$	-0.064 <sup>c</sup> (.035)	-0.007 (.032)	0.002 (.038)	0.190 <sup>a</sup> (.036)
$Colony_j$	0.100 <sup>a</sup> (.032)	0.466 <sup>a</sup> (.025)	0.141 <sup>a</sup> (.035)	0.442 <sup>a</sup> (.027)
$French_j$	0.213 <sup>a</sup> (.029)	0.991 <sup>a</sup> (.028)	0.188 <sup>a</sup> (.032)	1.015 <sup>a</sup> (.028)
N	23,553	23,553	23,553	23,553
R <sup>2</sup>	0.480	0.591	0.396	0.569

OLS estimates with year and industry dummies. Robust standard errors in parentheses.

Source : Crozet and Koenig, 2010

# A natural experiment : the euro drop ?

Graph 1 – Composition of growth in value of French manufacturing exports between 1998 and 2003 according to destination, in %



Source: A. Berthou and L. Fontagné (2008-a), *op. cit.*.

# Appendix

## How to derive the demand function

- Lagrangien :  $L = \left( \int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} - \mu \left( \int_{\Omega} p(\omega)q(\omega) - wL \right)$
- First order conditions :

$$\frac{\partial L}{\partial q(\omega)} = q(\omega)^{\frac{-1}{\sigma}} \left( \int_{\Omega} \omega^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{1}{\sigma-1}} - \mu p(\omega) = 0$$

$$\Leftrightarrow q(\omega)^{\frac{-1}{\sigma}} U^{\frac{1}{\sigma}} = \mu p(\omega)$$

$$\Leftrightarrow p(\omega)q(\omega) = U\mu^{-\sigma} p(\omega)^{1-\sigma}$$

- Integrate over  $\Omega$  :  $\int_{\Omega} p(\omega)q(\omega)d\omega = U\mu^{-\sigma} \int_{\Omega} p(\omega)^{1-\sigma} d\omega$  and  $U \equiv C = \left( \int_{\Omega} \omega^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} = U\mu^{-\sigma} \left( \int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{\sigma}{\sigma-1}}$
- Remember that  $wL \equiv PC$ , this gives :  $P = \left( \int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$

## How to derive the demand function (2)

- From :  $wL = U\mu^{-\sigma}P^{1-\sigma}$  and  $q(\omega)^{\frac{-1}{\sigma}} U^{\frac{1}{\sigma}} = \mu p(\omega)$ , one obtains the demand function :

$$q(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P}$$

- ⇒ Everything else being equal, a 1% rise in  $p(\omega)$  reduces demand  $q(\omega)$  by  $\sigma\%$  (ie  $\sigma$  measures the price-elasticity of demand)
- ⇒ The demand  $q(\omega)$  also depends on the consumer's purchasing power  $wL/P$
- $\frac{q(\omega)}{q(\omega')} = \left( \frac{p(\omega)}{p(\omega')} \right)^{-\sigma}$  : Increasing the relative price of the  $\omega$  variety by 1% reduces the relative demand for this variety by  $\sigma\%$  (elasticity of substitution)

## How to derive the optimal price

- Start from the firm's profit function :

$$\pi(\omega) = p(\omega)q(\omega) - w \left( f + \frac{q(\omega)}{\varphi} \right)$$

- Maximize with respect to price given demand function :

$q(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P}$  (Monopolistic competition  $\rightarrow$  The firm considers aggregate prices as given)

- $\Rightarrow$  First order condition :

$$\frac{\partial \pi(\omega)}{\partial p(\omega)} = P^{\sigma-1} wL \left[ (1 - \sigma) p^{-\sigma} + \frac{w}{\varphi} \sigma p^{-\sigma-1} \right] = 0$$

Or after rearranging :

$$p = \underbrace{\frac{\sigma}{\sigma-1}}_{\text{Mark-up}} \underbrace{\frac{w}{\varphi}}_{\text{Marginal cost}}$$

# Price indices in a two-country economy (Krugman)

- The price index now writes :

$$P = \left( \sum_{\omega \in H} p(\omega)^{1-\sigma} + \sum_{\omega \in F} (\tau p^*(\omega))^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- In the symmetric equilibrium,  $p(\omega) = p, \forall \omega \in H$  and  $p^*(\omega) = p^*, \forall \omega \in F$

$$\Rightarrow P = (np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma})^{\frac{1}{1-\sigma}}$$

$$\text{and } P^* = (n(\tau p)^{1-\sigma} + n^* p^{*1-\sigma})^{\frac{1}{1-\sigma}}$$

- Absent transportation costs ( $\tau = 1$ ), if marginal costs are equalized ( $p = p^*$ ), the two indices are equal whatever the relative size of the two countries :  $P = P^* = (2n)^{\frac{1}{1-\sigma}} p$ . Both countries have access to the same varieties in the same conditions.
- Both indices are lower than those in autarky, which are :  $P = n^{\frac{1}{1-\sigma}} p$  and  $P^* = n^{*\frac{1}{1-\sigma}} p^*$
- At given wages, opening up the economy has a positive impact on welfare ( $U = wL/P$ ). This comes from consumers' **preference for diversity**

## Wages in the Krugman model

- We have expressed prices  $p(\omega)$  and  $P$  as functions of nominal income  $wL$ , based on consumer's and firm's optimization
- $L$  is exogenous but  $w$  is endogenous
- In order to derive the wage level, you need to introduce one last equation : goods market equilibrium. Due to the Walras law, it is equivalent to rely on (i) the domestic market ( $wL = \sum_{\omega} w l(\omega)$ ); (ii) the foreign market ( $w^*L^* = \sum_{\omega} w^* l^*(\omega)$ ); (iii) the trade balance ( $X = X^*$ )
- We used the trade balance :

$$\lambda \times L \times L^* \times \left(\frac{\tau W}{P^*}\right)^{1-\sigma} \times w^* = \lambda \times L \times L^* \times \left(\frac{\tau W^*}{P}\right)^{1-\sigma} \times w$$

$$\Rightarrow \frac{w}{w^*} = \left(\frac{P}{P^*}\right)^{\frac{1-\sigma}{\sigma}}$$

$$\text{with } \left(\frac{P}{P^*}\right) = \frac{np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}}{n(\tau p)^{1-\sigma} + n^*p^{*1-\sigma}}$$

$$\Rightarrow \frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^*(\tau W^*)^{1-\sigma}}{L(\tau W)^{1-\sigma} + L^*w^{*1-\sigma}}\right)^{1/\sigma}$$

# Specialization (Helpman-Krugman)

- All firms producing differentiated goods produce the same quantity  $q$  that they sell at the same price  $p$  :

$$q = q^D + \tau q^X = \mu \left( \frac{p}{P} \right)^{-\sigma} \frac{wL}{P} + \tau \mu \left( \frac{\tau p}{P^*} \right)^{-\sigma} \frac{w^* L^*}{P^*}$$

- Replace price indices by their open-economy expressions :

$$q = \mu \frac{p^{-\sigma}}{n p^{1-\sigma} + n^* (\tau p^*)^{1-\sigma}} wL + \tau \mu \frac{(\tau p)^{-\sigma}}{n (\tau p)^{1-\sigma} + n^* p^{*1-\sigma}} w^* L^*$$

- Perfect labor mobility across sectors + Trade in homogeneous goods (same wage). Assume  $\varphi = \sigma/(\sigma - 1)$  so that  $p(\omega) = w = 1$  (normalization). The production of differentiated good writes, for each variety :

$$q = \mu \left( \frac{L}{n + n^* \tau^{1-\sigma}} + \frac{\tau^{1-\sigma} L^*}{n \tau^{1-\sigma} + n^*} \right) \quad q^* = \mu \left( \frac{\tau^{1-\sigma} L}{n + n^* \tau^{1-\sigma}} + \frac{L^*}{n \tau^{1-\sigma} + n^*} \right)$$

- Since  $q = q^*$ , we have :

$$\frac{L}{n + n^* \tau^{1-\sigma}} + \frac{\tau^{1-\sigma} L^*}{n \tau^{1-\sigma} + n^*} = \frac{\tau^{1-\sigma} L}{n + n^* \tau^{1-\sigma}} + \frac{L^*}{n \tau^{1-\sigma} + n^*}$$

or :

$$n \left( 1 - \frac{L}{L^*} \tau^{1-\sigma} \right) = n^* \left( \frac{L}{L^*} - \tau^{1-\sigma} \right)$$