Multi-Product Firms

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Topics in International Trade

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Motivation : Multi-Product Firms

- Melitz (2003) : Aggregate trade is dominated by large/high-productive firms
- Bernard et al (2014) : Large firms are also more likely to sell multiple products

$\Rightarrow$ Trade is dominated by multiple-product firms

- Their reaction to exogenous shocks (notably in terms of their product mix) is thus likely to matter substantially in the aggregate
- While the question has been extensively studied in the growth literature, little is known on the product-margin of international trade

Characteristics of Multi-Product Firms

TABLE 1
Summary statistics: cross-section 2005

Number of products exported	Number of firms		Value of exports		Average number of export destinations per firm	Average exports per firm-product-country (€1,000)	Average exports at the firm-product level (€1,000)	Average exports at the firm-country level (€1,000)
	N	% of total	Value (€1,000,000)	% of total				
Total exports								
1	8,596	34.05	4,487	2.08	1.58	331	522	331
2	3,401	13.47	4,157	1.93	3.07	317	611	398
3	2,026	8.02	3,952	1.83	4.44	301	650	440
4	1,392	5.51	4,032	1.87	5.42	327	724	534
5	1,102	4.36	6,764	3.13	6.73	506	1,228	912
6–10	3,187	12.62	21,947	10.17	9.56	326	903	720
11–20	2,483	9.83	38,655	17.92	12.85	375	1,058	1,211
21–30	1,068	4.23	31,483	14.59	15.94	391	1,179	1,849
31–50	899	3.56	28,693	13.30	18.66	261	819	1,710
>50	1,094	4.33	71,591	33.18	23.55	140	526	2,779
Total	25,248	100.00	215,761	100.00	6.73	230	741	1,270
Intrastat exports								
1	2,694	20.44	6,236	3.95	3.99	580	2,315	580
2	1,430	10.85	5,706	3.62	5.18	556	1,995	770
3	1,029	7.81	5,630	3.57	5.08	619	1,824	1,077
4	874	6.63	6,929	4.39	5.98	662	1,982	1,327
5	670	5.08	3,918	2.48	6.17	395	1,170	948
6–10	2,162	16.40	21,241	13.47	6.86	451	1,279	1,433
11–20	1,848	14.02	22,261	14.11	7.87	297	818	1,530
21–30	867	6.58	18,097	11.47	8.72	296	830	2,393
31–50	710	5.39	19,561	12.40	9.22	246	703	2,988
>50	893	6.78	48,135	30.52	10.10	132	428	5,336
Total	13,177	100.00	157,714	100.00	6.47	232	712	1,850
Extrastat exports								
1	8,674	44.35	1,353	2.33	1.24	125	156	125
2	3,289	16.81	1,050	1.81	2.22	113	160	144
3	1,764	9.02	1,005	1.73	3.33	118	190	171
4	1,212	6.20	1,029	1.77	4.44	121	212	191
5	872	4.46	813	1.40	5.52	99	186	169
6–10	1,920	9.82	5,213	8.98	8.55	159	362	317
11–20	1,070	5.47	16,254	28.00	13.56	441	1,051	1,120
21–30	333	1.70	13,638	23.49	19.79	599	1,662	2,070
31–50	252	1.29	8,183	14.10	25.90	281	840	1,254
>50	174	0.89	9,510	16.38	37.09	104	445	1,473
Total	19,560	100.00	58,047	100.00	4.33	225	587	686

Information on sample selection: See Data Appendix. A product is defined as an eight-digit Combined Nomenclature product.

Source : Bernard et al (2014), based on Belgian firm-level data

Characteristics of Multi-Product Firms

TABLE 2
Firm characteristics: Cross-section 2005

Number of products exported	ln(*Total factor productivity*)	ln(*Value added*)	ln(*Employment*)	ln(*Capital intensity*)
Total exports: All firms				
1	–0.35	12.74	1.69	10.20
2	–0.12	13.05	1.92	10.15
3	–0.21	13.27	2.11	10.28
4	–0.15	13.39	2.24	10.27
5	–0.14	13.48	2.28	10.24
6–10	–0.14	13.72	2.50	10.23
11–20	–0.07	14.02	2.76	10.17
21–30	–0.08	14.26	2.96	10.21
31–50	–0.03	14.64	3.33	10.10
>50	0.00	15.06	3.78	10.07

Information on sample selection: See Data Appendix. A product is defined as an eight-digit Combined Nomenclature product. All values are expressed in euros. Total factor productivity is calculated using the index number methodology (Caves *et al*., 1982). Employment is expressed in full-time equivalent units. Capital intensity is defined as tangible fixed assets per employee. Values reported are firm-level sample means, taken over all firms exporting the listed number of products.

Source : Bernard et al (2014), based on Belgian firm-level data

The product margin of trade

TABLE 3
Firm productivity and the margins of trade: 2005

	ln(*Value*$_f$)	ln(# *Countries*$_f$)	ln(# *Products*$_f$)	ln(*Density*$_f$)	ln(*Average value*$_f$)	ln(*Value*$_{fpc}$)
Using TFP to proxy for firm productivity						
Ln(*TFP*)	0.076**	0.022**	0.027**	−0.013**	0.040**	0.094***
	[0.035]	[0.011]	[0.012]	[0.007]	[0.020]	[0.035]
Fixed effects	Industry	Industry	Industry	Industry	Industry	Product-country
Clustering	No	No	No	No	No	Firm
Observations	16,278	16,278	16,278	16,278	16,278	684,860
R^2	0.241	0.194	0.143	0.139	0.221	0.405
Using labour productivity (value added per worker) to proxy for firm productivity						
ln(*VA/worker*)	0.762***	0.199***	0.173***	−0.101***	0.491***	0.309***
	[0.032]	[0.012]	[0.015]	[0.008]	[0.022]	[0.076]
Fixed effects	Industry	Industry	Industry	Industry	Industry	Product-country
Clustering	No	No	No	No	No	Firm
Observations	16,499	16,499	16,499	16,499	16,499	689,269
R^2	0.267	0.204	0.147	0.146	0.246	0.408

All results are obtained by running ordinary least squares regressions at the firm level, using data on total exports for 2005 (see Data Appendix for sample selection). The dependent variable used is reported at the top of each column. Reported values are coefficients [robust standard errors]. Significance levels: *** < 0.01; ** < 0.05. TFP, total factor productivity; VA, value added.

Source : Bernard et al (2014), based on Belgian firm-level data

$$\ln Value_f = \ln \sum_c \sum_p Value_{fcp} = \ln \#_c + \ln \#_p + \underbrace{\ln \frac{\#_{cp}}{\#_c \#_p}}_{Density} + \underbrace{\ln \frac{1}{\#_{cp} \sum_c \sum_p V_{fpc}}}_{Average\ Value_f}$$

The product margin of trade

TABLE 4
Within-firm productivity changes and the margins of trade

	ln(*Value*$_f$)	ln(# *Countries*$_f$)	ln(# *Products*$_f$)	ln(*Average value*$_f$)	ln(*Value*$_{fpc}$)
Annual differences					
Ln(*TFP*)	0.005**	0.002***	0.001*	0.002*	0.002
	[0.002]	[0.001]	[0.001]	[0.001]	[0.002]
Fixed effects	Firm, year	Firm, year	Firm, year	Firm, year	Firm-product-country +
Clustering	No	No	No	No	Year firm
Observations	135,077	135,077	135,077	135,077	4,686,642
R^2	0.890	0.890	0.880	0.870	0.890
Long differences (1998–2005)					
Ln(*TFP*)	0.032**	0.012**	0.018**	0.016**	0.073***
	[0.014]	[0.005]	[0.008]	[0.008]	[0.018]
Fixed effects	None	None	None	None	None
Clustering	No	No	No	No	Firm
Observations	8,648	8,648	8,648	8,648	165,594
R^2	0.002	0.002	0.002	0.001	0.002

All results are obtained by running regressions at the firm level or at the firm-product-country level (final column), using data on total exports between 1998 and 2005 (see Data Appendix for sample selection). The dependent variable used is reported at the top of each column. Reported values are coefficients [robust standard errors]. The top panel reports the results of a fixed effects regression (within-firm results). In the bottom panel both the dependent and independent variables are defined as long differences (i.e. the difference between 2005 and 1998). Significance levels: *** < 0.01; ** < 0.05; * < 0.1.

Source : Bernard et al (2014), based on Belgian firm-level data

$$\Delta \ln Value_f = \Delta \ln \sum_c \sum_p Value_{fcp} = \Delta \ln \#_c + \Delta \ln \#_p + \underbrace{\Delta \ln \frac{\#_{cp}}{\#_c \#_p}}_{Density} + \underbrace{\Delta \ln \frac{1}{\#_{cp} \sum_c \sum_p V_{fpc}}}_{Average\ Value_f}$$

Motivation : Why do we care ?

Multi-product firms matter for

- The structure and elasticity of trade
  - Bernard et al (2011) : Multiple products help explain a number of features of disaggregated trade data, including the skewness in export sales across products and the prositive correlation between # products, # destinations, and sales per destination
  - Firms react to tougher competition (Mayer et al, 2014) and trade liberalization (Bernard et al, 2011) by skewing their exports towards their best performing products
- The dynamics of industries (Lecture on this ?)
  - Anecdotal evidence that manufacturing firms increasingly grow through new products (eg financial services in the car industry)
  - Bernard et al (2010) : Product switching contributes to a reallocation of resources within firms toward their most efficient use

Modeling multi-product firms

- Supply-side economies of scale
- Heterogeneity in the ability of firms to produce different products
  - Eckel & Neary (2010) : Each firm has a core competence and faces increasing marginal costs in producing products further away from its core competence
  - Bernard et al (2011) : Preferences are heterogeneous regarding the different products produced by a firm
  - Mayer et al (2014) : Firms face a product ladder where productivity/quality declines discretely for each additional variety produced
  - Nocke & Yeaple (2006) : Firms differ in terms of organizational capability, which determines the rate at which the common marginal cost for each product rises with the number of products

A model of multi-product firms

Bernard, Redding and Schott (QJE, 2011)

A sketch of the model

- **Generalization of the Melitz (2003) model :**
  - Horizontal differentiation,
  - Monopolistic competition,
  - Sunk entry cost (before productivity is revealed)
  - Fixed per period cost per market **and per product**
- **Two degrees of heterogeneity :**
  - Heterogeneous productivity / ability $\rightarrow$ Selection across firms
  - Product attributes (idiosyncratic across products and markets) $\rightarrow$ Selection across products, within the firm.

    Product attributes are either common across markets within a firm (technology ?) or market-specific (perceived quality ?)

Hypotheses

- J countries $i = 1...J$ each endowed with L_i workers
- A mass one of products $k \in [0, 1]$
- 2-Layer CES preferences : Across products

$$U_j = \left[\int_0^1 q_{jk}^{\frac{\rho-1}{\rho}} dk \right]^{\frac{\rho}{\rho-1}}$$

and across vertically differentiated varieties within a product :

$$q_{jk} = \left[\sum_{i=1}^{J} \int_{\omega \in \Omega_{ijk}} \left[\lambda_{ijk}(\omega) q_{ijk}(\omega) \right]^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

with $\lambda_{ijk}(\omega)$ a random "product attribute" and $\sigma_k = \sigma > \rho$ elasticities of substitution

$\Rightarrow$ Price index :

$$P_{jk} = \left[\sum_{i=1}^{J} \int_{\omega \in \Omega_{ijk}} \left[\frac{p_{ijk}(\omega)}{\lambda_{ijk}(\omega)} \right]^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

Hypotheses

- An unbounded measure of potential firms face a sunk entry cost $f_{ei} > 0$
- After entry, a firm discovers
  - its productivity φ (drawn from a distribution $g_i(\varphi)$ with CDF $G_i(\varphi)$)
  - product attributes, $\lambda \in [0, \infty)$ drawn from a continuous distribution $z(\lambda)$ with CDF $Z(\lambda)$
- Two alternative specifications :
  - Common-product attributes : $\lambda_{jk}(\omega) = \lambda_k(\omega)\ \forall j$ (random technology)
  - Country-specific-product attributes : $\lambda_{ik}(\omega) \neq \lambda_{jk}(\omega)$ (random taste)
- Productivity draws and product attributes are independent across firms, independent of one another, independent across products and, in the country-specific-product-attribute case, independent across countries within a product (thus LLN will apply)

Hypotheses

- After uncertainty has been realized, firm decides which market(s) to serve
- A fixed cost per market $F_{ij} > 0$
- An additional fixed cost per market and product $f_{ij} > 0$
- A constant marginal cost of producing w_i/φ
- A transportation cost $\tau_{ij} > 1$

Implications

- Profit maximization, conditional on entry implies :

$$p_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega)) = \tau_{ij} \frac{\sigma}{\sigma - 1} \frac{w_i}{\varphi} = p_{ij}(\varphi(\omega))$$

- Optimal demand :

$$\begin{aligned} r_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega)) &= p_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega)) q_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega)) \\ &= \left(\frac{p_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega))}{\lambda_{ijk}(\omega) P_j} \right)^{1-\sigma} w_j L_j \end{aligned}$$

- Product-and-country-specific profits :

$$\pi_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega)) = \frac{r_{ijk}(\varphi(\omega), \lambda_{ijk}(\omega))}{\sigma} - w_i f_{ij}$$

Consequences of CES-MC

- Relative sales of any two firms selling the same product in a country :

$$\frac{r_{ijk}(\varphi,\lambda)}{r_{ijk}(\varphi',\lambda')} = \left(\frac{\varphi}{\varphi'}\right)^{\sigma-1}\left(\frac{\lambda}{\lambda'}\right)^{\sigma-1}$$

- Relative sales of a firm-product in any two countries :

$$\frac{r_{ijk}(\varphi,\lambda_{ijk}(\omega))}{r_{ij'k}(\varphi,\lambda_{ij'k}(\omega))} = \left(\frac{\tau_{ij}}{\tau_{ij'}}\right)^{1-\sigma}\left(\frac{\lambda_{ijk}(\omega)}{\lambda_{ij'k}(\omega)}\right)^{\sigma-1}\left(\frac{P_{jk}}{P_{j'k}}\right)^{\sigma-1}\frac{w_j L_j}{w_{j'} L_{j'}}$$

Selection : Across products within a firm

- Zero-profit cutoff for product attributes : $\lambda^*_{ijk}(\varphi)$ such that :

$$r_{ijk}(\varphi(\omega), \lambda^*_{ij}(\varphi(\omega))) = \sigma w_i f_{ij}$$

$\Rightarrow$ Within a firm, products with the worst attributes supplied only to the easiest markets (if any) :

$$\lambda^*_{ij}(\varphi(\omega)) = \frac{\tau_{ij}}{\tau_{ii}} \frac{P_i}{P_j} \left(\frac{f_{ij}}{f_{ii}} \frac{w_i L_i}{w_j L_j} \right)^{\frac{1}{\sigma-1}} \lambda^*_{ii}(\varphi(\omega))$$

In the country-product-specific-attribute case, a product can be exported without being sold domestically

$\Rightarrow$ Higher productivity firms have lower product cutoffs :

$$\lambda^*_{ijk}(\varphi(\omega)) = \left(\frac{\varphi^*_{ijk}}{\varphi(\omega)} \right) \lambda^*_{ijk}(\varphi^*_{ijk})$$

with φ^*_{ijk} the lowest productive firm exporting to country j

$\Rightarrow$ Markets with high φ^*_{ijk} or high $\lambda^*_{ijk}(\varphi^*_{ijk})$ are more competitive thus pushing each firm's product cutoff up

Selection : Across firms

- Total firm profits in market j :

$$\pi_{ij}(\varphi) = \int_{\lambda_{ij}^*(\varphi)}^{\infty} \left(\frac{r_{ij}(\varphi, \lambda)}{\sigma} - w_i f_{ij} \right) z(\lambda) d\lambda - w_i F_{ij}$$

- Low $\varphi \rightarrow$ High $\lambda_{ij}^*(\varphi) \rightarrow$ low proba of being able to sell a given product $\left[1 - Z(\lambda_{ij}^*(\varphi)\right]$

$\Rightarrow$ Low productivity firms

  - supply a smaller fraction of products to a given market
  - have lower expected profits for each product
  - are less likely to serve a given market

Selection : Across firms

- Zero-profit cutoff productivity : φ_{ij}^* such that :

$$\pi_{ij}(\varphi_{ij}^*) = 0$$

which implies $\lambda_{ij}^*(\varphi_{ij}^*)$ is implicitly given by :

$$\int_{\lambda_{ij}^*(\varphi_{ij}^*)}^{\infty} \left[\left(\frac{\lambda}{\lambda_{ij}^*(\varphi_{ij}^*)} \right)^{\sigma-1} - 1 \right] f_{ij} z(\lambda) d\lambda = F_{ij}$$

- Across markets :

$$\varphi_{ij}^* = \Gamma_{ijh} \varphi_{ih}^*, \quad \Gamma_{ijh} = \frac{\tau_{ij}}{\tau_{ih}} \frac{P_h}{P_j} \left(\frac{f_{ij}}{f_{ih}} \frac{w_h L_h}{w_j L_j} \right)^{\frac{1}{\sigma-1}} \frac{\lambda_{ih}^*(\varphi_{ih}^*)}{\lambda_{ij}^*(\varphi_{ij}^*)}$$

For sufficiently high fixed and variable trade costs, selection into exports : $\Gamma_{iji} > 1$

Resolution in GE

- Entry decisions : As in Melitz (2003)
- Use good and labor market equilibria to solve for equilibrium wages and price indices
- Solution with symmetric countries under general distributions of productivity and product attributes
- Solution with asymmetric countries assuming Pareto

The case with variable mark-ups

- Mayer et al (2014) propose an alternative (more elegant) model of multi-product firms
- A variation around Melitz and Ottaviano (2008) : Quasi-linear demand functions, Variable mark-ups, Exogenous wages
- Firms are endowed with a "core competency" which they produce at cost c and an increasing marginal cost for each additional variety

$$v(m, c) = \omega^{-m} c, \ \omega \in (0, 1)$$

- Solution with asymmetric countries assuming Pareto distribution of costs

(Additional) predictions

- More productive firms produce more products which they sell further away
- More competition induces
  - A selection of firms
  - A selection of products within firms
  - **A reallocation of resources towards the firm's better performing varieties ("pro-competitive effect")**

$\Rightarrow$ Increase in the firm's total productivity driven by the response of the firm's product mix ($\neq$ BRS, 2011)

Empirical evidence

Testable predictions

- Trade liberalization causes firms to drop their least-successful products $\rightarrow$ Within-firm "efficiency" gains (on top of across-firm reallocation)
- High variable trade costs $\rightarrow \downarrow$ number of exporting firms, $\downarrow$ number of products exported by each firm, and $\downarrow$ exports of a given product by a given firm, but ambiguous effect on average exports per firm and product
- Firms exporting many products also serve many export destinations and export more of a given product to a given destination

Empirical strategy

- Data : US Linked/Longitudinal Firm Trade Transaction database + US Census of Manufactures (1992-2004)
- A product = a 10-digit HS product / 5-digit SIC, partitioned into 4-digit SIC industries
- Use the Canada-US Free Trade Agreement as a natural experiment of trade liberalization (1988, heterogeneous across products)
- Firms' exposure to CUSFTA measured as the change in tariffs, in the industries in which it was active before the shock :

$$\Delta Tariff_f = \frac{\sum_i v_{fi}^{87} \Delta Tariff_i}{\sum_i v_{fi}^{87}}$$

with i a SIC industry and $\Delta Tariff_i$ the change in tariffs bw 1989 and 1992

Empirical strategy

- Dif-in-Dif strategy : Change in the number of products before and after trade liberalization, for firms experiencing above the median Canadian tariff reductions, in comparison with firms experiencing below the median tariff cut :

$$\#Products_{ft} = \beta Post_t \times Exposure_f + \eta_f + d_t + u_{ft}$$

  where $t = 1989/1992$ (equivalent to a specification in first differences)

- Model predicts $\beta < 0$ as more competition forces firms to reduce the scope of their production and concentrate on their most successful products

Dif-in-dif results

TABLE I

U.S. MANUFACTURING FIRM SCOPE DURING THE CANADA–U.S. FREE TRADE AGREEMENT

	[1]	[2]	[3]
Change in products	−0.059	−0.624	−0.572
	0.015	0.101	0.096
Change in entropy	0.011	0.156	0.153
	0.003	0.026	0.026
Firm observations	66,472	66,472	66,472
Major industry dummy variables	No	Yes	Yes
Log 1987 employment	No	No	Yes

Notes. Table reports mean difference in noted variable between surviving firms experiencing above- and below-median changes in Canadian export opportunities between 1987 and 1992. Each cell reports the mean difference and associated standard error from a separate OLS regression. Change in products refers to change in number of five-digit SIC categories produced in the United States. Change in entropy is defined in the text. Change in export opportunities refers to the output-weighted average change in Canadian tariffs across the four-digit SIC industries produced by the firm. Robust standard errors are clustered according to firms' main four-digit SIC industry. Additional covariates are included as noted.

Source : Bernard et al (2011), Entropy is a measure of sales' concentration : $\sum_k s_{fkt} \ln s_{fkt}$. A placebo exercise where the LHS variable is the change in products between 82 and 87 delivers non-significant results

Empirical strategy 2

- Test predictions on selection into exports using a gravity-type framework :

$$\ln Z_c = \alpha + \beta \ln Dist_c + \gamma \ln GDP_c + \varepsilon_c$$

- Intensive/extensive decomposition :

$$Value_c = \overline{Value}_c \#_c^{fp} = \overline{Value}_c \#_c^{f} \#_c^{p} d_c$$

- Model predicts :
  - That both the firm and product extensive margins depend on the market potential
  - That exports of a firm for a given product is decreasing in the difficulty of the market

Margins of trade

TABLE II

GRAVITY AND THE MARGINS OF U.S. EXPORTS

	$\ln(\text{Value}_c)$	$\ln(\text{Avg Exports}_c)$	$\ln(\text{Obs}_c)$	$\ln(\text{Firms}_c)$	$\ln(\text{Products}_c)$	$\ln(\text{Density}_c)$	$\ln(\text{Value}_{fpc})$
$\ln(\text{Distance}_c)$	−1.37	0.05	−1.43	−1.17	−1.10	0.84	−0.18
	0.17	0.10	0.17	0.15	0.15	0.13	0.080
$\ln(\text{GDP}_c)$	1.01	0.23	0.78	0.71	0.55	0.48	0.25
	0.04	0.02	0.04	0.03	0.03	0.03	0.020
Constant	7.82	6.03	1.80	0.52	3.48	−2.20	4.79
	1.83	1.07	1.81	1.59	1.55	1.37	0.64
Observations	175	175	175	175	175	175	1,878,532
Fixed effects	No	No	No	No	No	No	Firm-Product
R^2	0.82	0.37	0.75	0.76	0.68	0.66	0.70

Notes. Table reports results of OLS regressions of U.S. export value or its components on trading-partners' GDP and great-circle distance (in kilometers) from the United States. The first six columns are country-level regressions and final column is a firm-product-country level regression. Robust standard errors are noted below each coefficient; they are adjusted for clustering by country in the final column. Data are for 2002.

Source : Bernard et al (2011)

- Distance effect entirely attributable to the extensive margin
- Both the firm and the product margins matter
- Density increases with distance because firms do not cover the whole product scope
- Exports of a given firm/product decline with distance

Empirical strategy 3

- Model predicts :
  - That the participation of firms to trade and the number of products sold, conditional on exporting, are both correlated with the firm's size
  - That large firms also sell more at the intensive margin
- Correlate the number of exported products and the number of destinations served on two measures of firms' ability, total exports and estimated TFP
- Correlate the number of exported products and the number of destinations served on two measures of firms' "intensive" exports, exports of the firm's largest product and average exports per products

Margins of trade

TABLE III

CORRELATION OF U.S. FIRMS' EXTENSIVE AND INTENSIVE MARGINS

	ln(Products$_f$)					ln(Countries$_f$)				
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
ln(Size of Largest Product$_f$)	0.345 0.003					0.329 0.006				
ln (Size of 5th-Largest Product$_f$)		0.405 0.004								0.345 0.005
ln(Exports$_f$)			0.384 0.004					0.347 0.006		
ln(TFP$_f$)				0.071 0.022					0.076 0.022	
ln(Output$_f$/Worker$_f$)					0.474 0.019		0.426 0.020			
Constant	−2.300 0.061	0.405 0.004	−3.022 0.053	1.894 0.006	0.436 0.096	−2.714 0.078	−0.797 0.101	−3.141 0.072	1.292 0.006	−1.733 0.051
Observations	27,987	16,215	27,987	27,987	27,987	27,987	27,987	27,987	27,987	16,215
R^2	0.56	0.50	0.69	0.13	0.18	0.55	0.24	0.60	0.21	0.53

Notes. Table reports results of firm-level OLS regressions of the log number of 10-digit HS products exported by the firm, or log the number of destination countries served by the firm, on noted covariates. All regressions include dummies for firms' main four-digit SIC industry, and robust standard errors are clustered on this dimension of the data. Results in columns 2 and 7 are restricted to firms exporting at least five products. Data are for 1997.

Source : Bernard et al (2011)

Skewness of exports, within firms

TABLE IV

DISTRIBUTION OF FIRM EXPORTS ACROSS PRODUCTS, 2002

Rank	All Exports	Products Exported to Canada	HS 84-85 Products Exported to Canada
1	49.0	47.4	47.9
2	18.6	19.4	19.3
3	10.5	11.1	11.0
4	6.7	7.0	7.0
5	4.6	4.8	4.7
6	3.4	3.4	3.3
7	2.5	2.5	2.4
8	1.9	1.9	1.8
9	1.5	1.5	1.4
10	1.1	1.1	1.1

Notes. Columns report the mean percent of firm exports represented by the product with the noted rank (from high to low) across firms exporting 10, 10-digit HS products in 2002. Second and third columns restrict observations to firms exporting 10 products to Canada, and firms exporting 10 Machinery and Electrical products (HS 84-85) to Canada, respectively. Sample sizes across the three columns are 1641, 983, and 322 firms, respectively.

Source : Bernard et al (2011)

Skewness of exports, within firms

TABLE 5—SKEWNESS MEASURES FOR EXPORT SALES OF ALL PRODUCTS

	(1)	(2)	(3)	(4)	(5)	(6)
ln GDP	0.141***	0.019***	0.047***	0.052***	0.047***	0.041***
	(0.010)	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)
ln supply potential	0.125***	0.016***	0.037***	0.033***	0.023***	0.031***
	(0.023)	(0.002)	(0.004)	(0.004)	(0.004)	(0.004)
ln freeness of trade	0.096***	0.007*	0.021**	0.032**	0.045**	0.021**
	(0.036)	(0.004)	(0.009)	(0.013)	(0.022)	(0.009)
ln GDP per cap						0.013**
						(0.005)
Dep. Var.	s.d. ln x	herf	theil	theil	theil	theil
Destination GDP/cap	all	all	all	top 50%	top 20%	all
Observations	82090	82090	82090	73029	57076	82090
Within R^2	0.107	0.164	0.359	0.356	0.341	0.359

Note: All columns use Wooldridge's (2006) procedure: country-specific random effects on firm-demeaned data, with a robust covariance matrix estimation. Standard errors in parentheses. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All columns include a cubic polynomial of the number of products exported by the firm to the country (also included in the within R^2).

Source : Mayer et al (2014), based on French data

Conclusions

- The "extensive" margin of trade is broader than you think
  - Entry/exit of firms within a market
  - Entry/exit of products within a firm and a market
  - Changes in the number of clients a firm serves in a destination within a market
  - ...
- The dimensions through which efficiency gains can happen are also multiple $\rightarrow$ Gains from trade might be larger than you think (See Melitz & Redding)
- Drawback : Taking these dimensions into account requires extending the dimensionality of the "heterogeneity". As long as those dimensions are not observable and somewhat correlated, it is not clear how much we learn from this

References

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Demand function

- Proba that a consumer from d chooses variety ω is

$$\begin{aligned}
& P[V(\omega) \geq V(\omega') \forall \omega' \neq \omega] \\
= \; & P[\theta_d q(\omega) - \delta_d^{-1} p_d(\omega) + \varepsilon \geq \theta_d q(\omega') - \delta_d^{-1} p_d(\omega') + \varepsilon' \forall \omega' \neq \omega] \\
= \; & P[\theta_d q(\omega) - \delta_d^{-1} p_d(\omega) - \theta_d q(\omega') + \delta_d^{-1} p_d(\omega') \geq \varepsilon' - \varepsilon \forall \omega' \neq \omega] \\
= \; & \int_{-\infty}^{\infty} f(x) \prod_{\omega' \neq \omega} F(\theta_d q(\omega) - \delta_d^{-1} p_d(\omega) - \theta_d q(\omega') + \delta_d^{-1} p_d(\omega') + x) dx
\end{aligned}$$

Using the change of variable $\alpha = exp\left[-\left(\frac{x}{\mu} + \gamma\right)\right]$ and $y(\omega) = exp\left(\frac{\theta_d q(\omega) - \delta_d^{-1} p_d(\omega)}{\mu}\right)$, this implies :

$$\begin{aligned}
P[V(\omega) \geq V(\omega') \forall \omega' \neq \omega] &= \int_0^{\infty} exp(-\alpha) \prod_{\omega' \neq \omega} \left[exp\left(-\frac{\alpha y(\omega')}{y(\omega)}\right)\right] d\alpha \\
&= \int_0^{\infty} exp\left[-\alpha \left(\int_{\Omega_d} \frac{y(\omega')}{y(\omega)} d\omega'\right)\right] d\alpha \\
&= \frac{y(\omega)}{\int_{\Omega_d} y(\omega) d\omega}
\end{aligned}$$