#### Trade Networks

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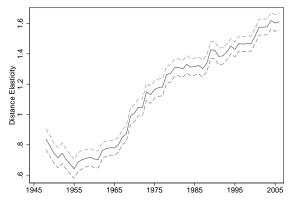
Topics in International Trade University Paris-Saclay Master in Economics, 2nd year

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#### Motivation : Trade Frictions

- Samuelson (1954) and Krugman (1980) : Key importance of frictions in shaping the patterns of international trade and relative prices
- Crude formalization : "Iceberg" trade costs (+ eventually a fixed cost) which encompass many different trade "barriers" eg. trade policy, transportation costs, cost of trading with partners with a different cultural background, under different legal structures, etc.
- Rauch (1999) : Potential role of informational barriers to explain the "increasing cost of distance"  $\rightarrow$  Difficulty to locate potential partners and uncertainty on contracts' enforceability, especially when trade relationships become more "complex", eg within GVCs
- Rauch (2001) and Rauch and Trindade (2002) : Impact of business and social networks in facilitating trade

#### The rising cost of distance



Source : Author's calculation based on data in Head et al. (2010). Plain line is the absolute value of the distance coefficient estimated using :

$$lnX_{ij} = FE_i + FE_j + lndist_{ij} + \chi Controles_{ij} + \varepsilon_{ij}$$

Dotted lines identify the confidence interval at 5%.

Variable	Org.	Ref.	Dif.	Org.	Ref.	Dif.
Intercept	-44.502	-21.505	-16.673	-42.373	-19.039	-13.236
	(3.904)	(2.862)	(2.640)	(3.932)	(2.875)	(2.648)
Threshold (\$US thous.)	140.343*	117.709*	94.672*	140.141*	117.837*	95.607ª
	(18.900)	(14.975)	(15.616)	(18.882)	(14.970)	(15.724)
$\ln (GNP_iGNP_j)$ (1980)	1.077ª	0.912 <sup>a</sup>	0.903ª	1.074ª	0.907ª	0.897 <sup>a</sup>
	(0.041)	(0.028)	(0.027)	(0.041)	(0.028)	(0.027)
$\ln (PGNP_iPGNP_i)$ (1980)	0.382 <sup>a</sup>	0.494*	0.535*	0.367ª	0.476 <sup>a</sup>	0.510ª
	(0.051)	(0.036)	(0.036)	(0.051)	(0.036)	(0.036)
In (DISTANCE)	-1.416 <sup>a</sup>	-1.114ª	-0.858 <sup>a</sup>	$-1.410^{a}$	-1.107 <sup>a</sup>	-0.847 <sup>a</sup>
	(0.111)	(0.086)	(0.082)	(0.111)	(0.086)	(0.082)
In (REMOTE)	2.005 <sup>a</sup>	0.693ª	0.317 <sup>b</sup>	1.898 <sup>a</sup>	0.570 <sup>a</sup>	0.146
	(0.222)	(0.172)	(0.159)	(0.222)	(0.172)	(0.159)
ADJACENT	0.046	0.516 <sup>c</sup>	0.643 <sup>b</sup>	0.075	0.549 <sup>b</sup>	0.689 <sup>b</sup>
	(0.353)	(0.272)	(0.274)	(0.354)	(0.274)	(0.278)
EEC	-0.351	-0.060	-0.020	-0.344	-0.051	-0.006
	(0.228)	(0.160)	(0.148)	(0.227)	(0.159)	(0.147)
EFTA	-0.642	0.232	0.434 <sup>b</sup>	-0.643	0.232	0.434 <sup>b</sup>
	(0.410)	(0.219)	(0.219)	(0.409)	(0.218)	(0.216)
LANGUAGE	0.092	0.047	-0.382	0.201	0.172	-0.211
	(0.470)	(0.368)	(0.275)	(0.473)	(0.371)	(0.279)
COLOTIE	0.631ª	0.933ª	1.259ª	0.592 <sup>b</sup>	0.888*	1.198*
	(0.234)	(0.175)	(0.166)	(0.234)	(0.174)	(0.163)
CHINSHARE	3.696*	4.796*	5.963ª			
	(1.033)	(0.849)	(0.880)			
CHINSHARE * (1 - TWO800NE)	_			277.283*	327.196*	456.104ª
				(79.553)	(48.744)	(56.349)
CHINSHARE * TWO800NE	_	_	_	3.680ª	4.776*	5.935*
				(1.039)	(0.858)	(0.893)
Log likelihood	-16262.2	-16777.1	-18431.9	-16258.9	-16769.1	-18414.8

TABLE 3.—DEPENDENT VARIABLE: LOG OF 1980 BILATERAL TRADE IN ORGANIZED EXCHANGE, REFERENCE PRICED, AND DIFFERENTIATED COMMODITIES (CONSERVATIVE AGGREGATION)

Maximum likelihood estimation of threshold Tobit model.

Eicker-White standard errors in parentheses. Number of observations = 1595.

\* Significant at 1% level.

<sup>b</sup> Significant at 5% level.

<sup>c</sup> Significant at 10% level.

Source : Rauch & Trindade (2002).

- See Rauch (2000)
- Social or coethnic networks are communities of individuals or businesses that share a demographic attribute such as ethnicity or religion
- Business networks are sets of firms that are integrated neither completely nor barely at all and where the lineages of the members can often be traced back to a founding family or small number of allied families (eg Japanese *keiretsu*)
- Less easily observed networks include "alumniis of ENSAE", "former employees of IBM", etc.

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- · International networks can be favored by
  - migrations (Rauch and Trindade, 2002),
  - foreign direct investment (Mayer et al, 2010)
  - Indirect evidence : Impact of past migrations / FDI flows on the probability to export, do FDI, etc
- Chaney (2014) : More "statistical" view of networks
  - Trading with foreign partners should increase the probability that you meet with new partners there, or closeby

 $\Rightarrow$  Distribution of trade should inherit the network property

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- Impact of such networks :
  - Repeated exchanges that help sustain colusions,
  - Knowledge of each others' characteristics,
  - Access to your network's network
- $\Rightarrow$  Mitigate informational barriers

# Motivation : Why do we care?

Networks in international markets might matter for

- The patterns of international trade and heterogeneous export behaviors (Chaney, 2014)
- The dynamics of trade and, more specifically, the persistence of international trade relationships
  - Under informational frictions, individuals would prefer long-term, stable and direct relationships

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• (Informational frictions) The prevalence of trade intermediaries

#### A model of trade networks

# Chaney (AER, 2014)

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# A sketch of the model

- A dynamic model of trade with informational frictions
- Potential exporters meet with foreign partners in two distinct ways
  - Direct search (a geographically biased random search)
  - Remote search from already acquired foreign networks (a geographically biased random search from foreign destinations)
- Testable implications :
  - A firm which exports to country *a* in *t* is more likely to enter location *b* geographically close to *a* in *t* + 1 (biased network expansion ≠ Melitz-Chaney in which there is a strict hierarchy of foreign countries)
  - · Fat-tailed distribution for the number of foreign contacts across firms
  - Geographic distance of exports increases with the number of foreign contacts

# Motivating stylized facts

Use a probit estimator and firm-level panel export data to show that

- The probability to enter a new market is increasing in the number of markets which the firm already serves
- The probability to enter a specific market is decreasing in the distance between this market and the firm's existing portfolio of markets
- The probability to enter a specific market is increasing in the growth rate of exports between the firm's existing portfolio of markets and this country
- Every year, a firm has a 60% chance of exiting a country which it is currently serving
- ⇒ Firms follow a history-dependent process which governs their gradual entry into foreign markets

#### Hypotheses

- S a discrete set of locations. Time is discrete
- In each location  $x \in S$ , a finite number of firms (grows at rate  $\gamma$ )
- Model focuses on the **extensive margin of trade under search frictions**
- Firm *i* of age *t* has  $m_{i,t} = \sum_{x \in S} f_{i,t}(x)$  consumers, where  $f_{i,t}(x)$  is the number of consumers in location *x*
- Every period, a firm acquires new consumers :
  - from a local search :  $\tilde{\gamma\mu}$  (random) new consumers, located randomly according to g :

$$P[\mathbb{1}(\tilde{x}_{i,k_0}=x)]=g(0,x)$$

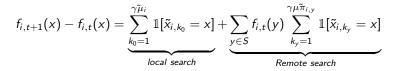
 $k_0$  a consumer met from x = 0

• from remote search : For each existing consumer in y,  $\gamma \tilde{\mu} \pi$  (random) new consumers ( $\pi \ge 0$ ), located randomly according to g

$$P[\mathbb{1}(\tilde{x}_{i,k_y}=x)]=g(y,x)$$

#### Firm-level dynamics

• Dynamic evolution of the network :



with the initial condition  $f_{i,0}(x) = 0 \ \forall x \in S$ 

 $\Rightarrow$  History dependent path, Heterogeneity across firms

### Aggregate dynamics

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• Suppose there are sufficiently many firms : Given N firms of age t located at 0, the average number of contacts in x is

$$f_t^N(x) = \frac{\sum_{i=1}^N f_{i,t}(x)}{N}$$

and  $\lim_{N\to\infty} f_t^N(x) = f_t(x)$ 

• Dynamics of the cohort's network :

$$f_{t+1}(x) - f_t(x) = \gamma \mu g(0, x) + \gamma \mu \pi \sum_{y \in S} f_t(y) g(y, x)$$

◀ Proof

### Aggregate dynamics

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• Number of consumers :

$$m_{t+1} - m_t = \gamma \mu + \gamma \mu \pi m_t$$

 $m_0 = 0$ 

• Under a mean-field approximation (number of a firm's contacts evolves as the population average), fraction of firms with fewer than *m* consumers (over all cohorts) :

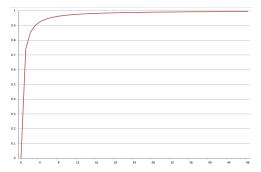
$$F(m) = 1 - \left(\frac{1}{1+\pi m}\right)^{rac{\ln(1+\gamma)}{\ln(1+\gamma\mu\pi)}}$$

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# Aggregate dynamics

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This graph represents F(m) as a function of m when  $\gamma = .02$ ,  $\pi = 2.4$  and  $\mu = 0.38$ .

- Lower tail close to an exponential distribution (mostly local search matters)
- Upper tail asymptotes to a Pareto distribution (mostly remote search matters)

### Geography of Trade Networks

Assume further,

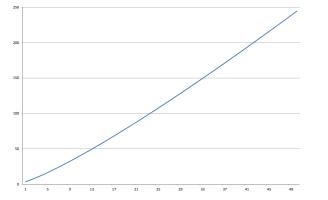
• 
$$S = \mathbb{Z}$$

- g(y, x) only depends on |x y|
- g(|x-y|) has a finite second moment  $(\Delta_g)$
- $\Rightarrow$   $f_t$  admits a closed-form solution (see Appendix in the paper)
- Under the mean-field approximation, the average squared distance from a firm's consumers :

$$\Delta(m) = \frac{\gamma \mu \pi}{(1 + \gamma \mu \pi) \ln(1 + \gamma \mu \pi)} \left(1 + \frac{1}{\pi m}\right) \ln(1 + \pi m) \Delta_g$$

which is increasing in the number of consumers m (because of remote search :  $\Delta(m) = \Delta_g$  if  $\pi \to 0$ ) Note : Intuition straightforward, Proof uses Fourier transformation to manipulate convolution products

#### Geography of Trade Networks



This graph represents  $\Delta(m)/\Delta_g$  as a function of *m* when  $\gamma = .02$ ,  $\pi = 2.4$  and  $\mu = 0.38$ .

# Geography of Trade Networks

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- Because of remote search, the acquisition of additional networks is biased towards more remote and more dispersed consumers
- While this is true on average, firms within a cohort exhibit a lot of heterogeneity (history-dependent path)
- Over time, the heterogeneity tends to increase, within a cohort (up to the point when all firms serve all consumers in the world)
- Results on the geography of networks under S = Z seem to be a good approximation of the geography simulated for S ≠ Z

### **Empirics on trade networks**

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## Empirics on Trade Networks

- Difficulties for testing such theories due to the absence of good data
  - Chaney (2014) is a model of consumers' acquisition but existing data are at the firm×destination-level, most of the time  $\rightarrow$  Test based on the acquisition of new destination markets (and thus m < 200)
  - Alternative : Indirect evidence based on "observed" networks (eg Rauch and Trindade, 2002)
  - Recently : Data on firm-to-firm trade have been made available to researchers → Better-suited to test network theories. Avenue for future research on networks and their determinants (beyond geography)
- Chaney (2014) :
  - Uses French firm-to-destination data, over 1986-1992 and
  - a SMM to bridge the gap between a micro-model (firms to contacts) and macro-data (firms to countries)

# Chaney (2014) : Testable predictions

- 1. The distribution of the number of consumers across firms is a mixture of an exponential and a Pareto distribution
  - Parametrized by  $\mu$  (# new consumers acquired each period via local search) and  $\pi$  (efficiency of remote search)
- 2. Average distance from consumers is increasing in the existing number of consumers
  - Parametrized by  $\pi$  (efficiency of remote search relative to direct search)

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# Chaney (2014) : SMM

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• Simulated equation :

$$f_{i,t+1}(x) - f_{i,t}(x) = \sum_{k_0=1}^{\gamma \tilde{\mu}_i} \mathbb{1}[\tilde{x}_{i,k_0} = x] + \sum_{y \in S} f_{i,t}(y) \sum_{k_y=1}^{\gamma \mu \tilde{\pi}_{i,y}} \mathbb{1}[\tilde{x}_{i,k_y} = x]$$

Functional form assumptions :

$$g(y,x) = \alpha_{\lambda,y} GDP_x e^{-||x-y||/\lambda}$$

where  $\alpha_{\lambda,y}=1/\sum_x \textit{GDP}_x e^{-||x-y||/\lambda}$  and  $\lambda$  scaling the geographic dispersion of new contacts

• Calibrated parameters :

$$\gamma = .02$$

Vector of estimated parameters :

$$\Theta = (\mu, \pi, \lambda)$$

# Chaney (2014) : SMM

- Given Θ = (μ, π, λ), simulate 360 successive cohorts of French firms of increasing size (20 × 1.02<sup>t</sup>) and store the random networks of consumers, over time
- Por each link, draw a destination country in g(c, c') where c is the origin country and c' the destination country
- 3 Iterate on step 2 to best fit 120 moments in the data :
  - Fraction of firms exporting to 1, 2, ..., 69 and 70 or more countries
     (f(M) = F(M+1) - F(M) in the model, where M counts countries
     instead of consumers)
  - Average squared distance among firms that export to 1, 2, ..., 49, 50 and more countries (Δ(M) in the model) :

$$\Delta(M) = \frac{\sum_{i \in \Xi(M)} \sum_{c} Dist_{France,c}^{2} \frac{1}{GDP_{c}} \mathbb{1}[export_{i,c} > 0]}{\sum_{i \in \Xi(M)} \sum_{c} \frac{1}{GDP_{c}} \mathbb{1}[export_{i,c} > 0]}$$

# Chaney (2014) : SMM

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#### 4 Define

$$\mathbf{y}(\Theta) = \mathbf{k} - \mathbf{\hat{k}}(\Theta)$$

a vector of deviations between te actual and simulated moments. Under the moment condition that  $E[\mathbf{y}(\Theta_0)] = 0$  for the true value  $\Theta_0$ , the set of parameters minimizes the weighted deviations between actual and simulated moments :

$$\hat{\Theta} = arg \ min_{\Theta} \{ \mathbf{y}(\Theta)' \mathbf{W} \mathbf{y}(\Theta) \}$$

where  $\mathbf{W}$  is a weight matrix

# Chaney (2014) : Estimated parameters

	(1986)	(1987)	(1988)	(1989)	(1990)	(1991)	(1992)
π	2.420 (0.187)	2.495 (0.114)	2.479 (0.150)	2.499 (0.066)	2.574 (0.114)	2.633 (0.130)	2.401 (0.200)
μ	0.371 (0.022)	0.368 (0.013)	$   \begin{array}{c}     0.384 \\     (0.021)   \end{array} $	0.362 (0.010)	0.357 (0.013)	0.338 (0.014)	0.384 (0.027)
Parameter	r  for  g(  x - y  ) =	$=\frac{1}{\lambda}e^{-  x-y  /\lambda}$					
λ	3.419 (0.131)	3.398 (0.145)	3.448 (0.130)	2.906 (0.403)	3.515 (0.177)	3.418 (0.132)	3.513 (0.135)

TABLE 2—DIRECT SEARCH, REMOTE SEARCH, AND GEOGRAPHY (SMM estimates)

Notes: This table presents the SMM estimates of  $\mu$ ,  $\pi$ , and  $\lambda$ . The parameters  $\mu$  and  $\pi$  govern the acquisition of the number of new consumers, while the parameter  $\lambda$  governs the geographic location of those consumers. Data: all French exporters, 1986–1992. Bootstrapped standard errors are in parentheses. All coefficients are statistically different from zero at the 1 percent level of significance.

Source : Chaney (2014)

- Remote search is more than twice as important as direct search for a firm with a single existing contact
- Relative importance of remote search growth as *m* gets larger (eg accounts for 90% of new contacts at the sample mean)

### Chaney (2014) : Distribution of contacts

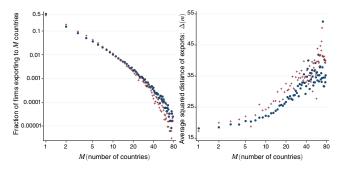


FIGURE 3. THE NUMBER AND GEOGRAPHY OF EXPORTS (SMM estimates)

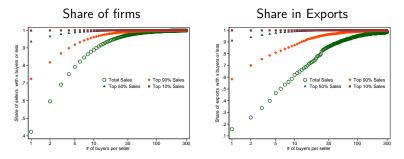
Notes: Left panel: fraction of firms that export to *M* different countries. Right panel: average squared distance to a firm's export destinations, among firms exporting to *M* destinations, as defined in equation (8); distances are calculated in thousands of kilometers. Dots: data, all French exporters in 1992. Plus signs: simulated data;  $\pi = 2.401$  (0.200),  $\mu = 0.384$  (0.027) and  $\lambda = 3.513$  (0.135) are estimated by simulated method of moments.

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Source : Chaney (2014)

#### Firm-to-firm data : Stock of consumers



Source : Author's calculations. Data covering the universe of French firms and their exports in the EU15 (data for 2007).

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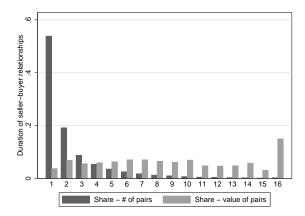
#### Firm-to-firm data : Stock of consumers

	ln # buyers			ln Herfindahl			
	(1)	(2)	(3)	(4)	(5)	(6)	
In value of exports	$0.22^{***}$	0.21***	$0.28^{***}$	-0.08***	-0.10***	-0.13***	
	(0.022)	(0.010)	(0.015)	(0.013)	(0.006)	(0.010)	
(ln value of exports) <sup>2</sup>	-0.01***	-0.01***	-0.01***	$0.01^{***}$	$0.01^{***}$	0.01***	
	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	
ln experience in dest.	0.11***	$0.34^{***}$	0.1 3***	- 0.06***	-0.22***	-0.10***	
-	(0.008)	(0.020)	(0.019)	(0.005)	(0.012)	(0.013)	
ln # products	0.40***	$0.74^{***}$	$0.53^{***}$				
	(0.013)	(0.020)	(0.023)				
In Herfindahl ac. products				$0.27^{***}$	$0.39^{***}$	$0.35^{***}$	
-				(0.010)	(0.014)	(0.014)	
1 = 1 if HQ in dest.	$-0.19^{***}$	- 0.01	-0.02	0.16***	0.02	0.04***	
-	(0.033)	(0.032)	(0.024)	(0.018)	(0.015)	(0.014)	
1 = 1 if affiliates in dest.	-0.19***	- 0.04	-0.18***	0.1 3***	0.03	0.13***	
	(0.052)	(0.086)	(0.060)	(0.034)	(0.051)	(0.040)	
In potential # of buyers	0.04***	0.00	0.00	. ,	. ,		
	(0.006)	(0.003)	(0.004)				
ln potential Herfindahl	. ,	. ,	. ,	0.03***	$0.09^{***}$	$0.03^{***}$	
-				(0.004)	(0.014)	(0.006)	
FE $Sect \times dest$ .	Yes	No	Yes	Yes	No	Yes	
FE Firm	No	Yes	Yes	No	Ye s	Yes	
# obs.	158, 239	158, 239	158, 239	158, 239	158,239	158,239	
$R^2$	0.184	0.294	0.676	0.100	0.139	0.556	

Table 3: Determinants of firm-level diversification within a country

Notes: Standard errors in parentheses clustered in the destination × sector dimension with \*\*\*, \*\* and \* respectively denoting significance at the 1, 5 and 10% levels. "In potential # of buyers" is the log of a (weighted) average of the number of firms buying at least one variety (whatever the exporter buying it) each nc8 sector in which the exporter is active. "In potential Herifindahl" is the log of the Herifindahl that the firm would display if it was serving each potential buyer of its nc8 products in proportion of their total purchases.

# Mismatch ? Durations of firm-to-firm relationships



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Source : Author's calculations

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### Details on Rauch & Trindade

• Estimation using Eaton & Tamura (1994) threshold Tobit model where "Threshold" is the minimum threshold value before strictly positive values are observed :

$$\ln(a_k + X_{ijk}) = max[\beta_k VAR_{ijk} + \varepsilon_{ijk}, \ln a_k]$$

- Important explanatory variables :
  - "REMOTE" is a measure of how "remote" each country is from the rest of the world (product of the weighted sum of country *i*'s distances from all other countries and the same weighted sum for country *j*)
  - "CHINSHARE" is the product of the ethnic Chinese population shares for countries *i* and *j*
  - "TWO80ONE" is a dummy equal to one if the populations of both *i* and *j* are at least 1% Chinese in 1980
- Goods are separated into three categories k : "Org." organized exchanges, "Ref" goods sold on markets with reference prices and "Dif" differentiated products

# Details on the dynamics of a cohort's network

$$f_{t+1}^{N}(x) - f_{t}^{N}(x) = \frac{\sum_{i=1}^{N} (f_{i,t+1}(x) - f_{i,t}(x))}{N}$$

$$= \frac{\sum_{i=1}^{N} \sum_{k=1}^{\gamma \tilde{\mu}_{i}} 1(\tilde{x}_{i,k} = x)}{N} + \frac{\sum_{i=1}^{N} \sum_{y \in S} \frac{m_{i,t}f_{i,t}(y)}{m_{i,t}} \sum_{k_{y}=1}^{\gamma \mu \tilde{\pi}_{i,y}} 1(\tilde{x}_{i,k_{y}} = x)}{N}$$

$$= \frac{\sum_{i=1}^{N} \sum_{k=1}^{\gamma \tilde{\mu}_{i}} 1(\tilde{x}_{i,k} = x)}{N} + m_{t} \sum_{y \in S} \frac{\sum_{i=1}^{N} \sum_{k_{y}=1}^{\gamma \mu \tilde{\pi}_{i,y}} g_{i,t}(x) 1(\tilde{x}_{i,k_{y}} = x)}{N}$$

$$\sim \gamma \mu g(0, x) + m_{t} \sum_{y \in S} \gamma \mu \pi h_{t}(y, y, x)$$

with  $g_{i,t} = f_{i,t}(x)/m_{i,t}$  and  $h_t(y, y, x) = g_t(y)g(y, x) = \frac{f_t(y)}{m_t}g(y, x)$  the joint probability distribution of "a random draw from all firms' contacts at t is in y and a random new search from y is in x" (Back to model)

 $\rightarrow N \rightarrow$ 

# Details on the dynamics of a cohort's network

• From the difference equation  $m_{t+1} - m_t = \gamma \mu + \gamma \mu \pi m_t$  :

$$m_t = \frac{1}{\pi}[(1+\gamma\mu\pi)^t - 1]$$

• Thus the age of a firm as a function of its number of contacts (use a mean-field approximation)

$$t(m) = \frac{\ln(1+\pi m)}{\ln(1+\gamma\mu\pi)}$$

• And the fraction of firms with more than m contacts (older than t(m)) :

$$1 - F(m) = (1 + \gamma)^{-t(m)} = (1 + \pi m)^{-\ln(1 + \gamma)/\ln(1 + \gamma \mu \pi)}$$

Back to model