Lecture 4: The Heckscher-Ohlin Model With Many Goods and Factors

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Outline of Lecture 4

1. When FPE Holds: the HOV Theorem
   - The HOV Theorem

2. When Does FPE Hold?
   - The 'Even' Case
   - More Goods Than Factors
   - More Factors than Goods

Introduction: Why Is It Difficult To Generalize HOS?

- We defined a free trade equilibrium as:

$$p = A(\omega^c)'\omega^c, c = H, F \tag{ZP}$$

$$y^H + y^F = \alpha(p)(\omega^H V^H + \omega^F V^F) \tag{GM}$$

$$V^c = A(\omega^c)y^c, c = H, F \tag{FE}$$

- With an equal number of goods and factors A is a square matrix.
- With an unequal number of goods and factors some variables are indeterminate:
  - with more goods than factors production is indeterminate: within the FPE set many output vectors y satisfy (FE)
  - with more factors than goods the FPE set is 'flat' (fewer dimensions than factor space) and of measure zero

The Heckscher-Ohlin-Vanek (HOV) model

- We start by deriving a prediction on trade patterns **when FPE is assumed to hold**.
- Assumptions:
  - $c = 1, ..., C$ countries; $i = 1, ..., N$ goods; $v = 1, ..., V$ factors
  - identical technologies
  - identical, homothetic preferences
  - FPE holds
- Denote net exports by T^c. Define the *factor content of trade* as:

$$F^c_{(V,1)} = A_{(V,N)}.T^c_{(N,1)}$$

- The main testable proposition of HOV relates the factor content of a country's net exports to its factor abundance.

The HOV Theorem

- By definition: $AT^c = Ay^c - Ax^c$

  where x^c is consumption in country c

- With homothetic preferences x^c is proportional to world demand x^w. Let $s^c = \frac{x^c}{x^w}$.
- From the world (FE) conditions, since technologies are identical and since world production equals world consumption:

$$Ax^c = s^c Ax^w = s^c Ay^w = s^c V^w$$

- From each country's (FE) condition Ay^c can be substituted by V^c.

Theorem (Heckscher-Ohlin-Vanek)

At free trade, a country's net exports are intensive in factors in which it is disproportionately endowed, or:

$$\forall c, F^c = V^c - s^c V^w$$

- s^c represents the share of country c in the world's GDP.
- Factor v is abundant in country c if c has more than its share (s^c) of the world endowment of v.

Corollary (Leamer, 1980)

If country c has a greater share of the world's capital than of the world's labor, i.e. $\frac{K^c}{K^w} > \frac{L^c}{L^w}$*, then the capital content of production exceeds the capital content of consumption:*

$$\frac{K^c}{L^c} > \frac{K^c - F_K^c}{L^c - F_L^c}$$

- From the HOV result it follows that $K^c - F_K^c = s^c K^w$.
- As in the 2-factor Edgeworth box diagram, net exports embody net exports of factor services.
- This prediction can be tested (see next lecture).

FPE In The 'Even' Case

- Does FPE hold in the 'even' case of N goods and factors?
- With N goods and factors, we can solve for the N factor prices in the N (ZP) equations.
- The solution is unique under a generalised no-FIR condition guarantees (Nikaido, 1972, see Feenstra p68).
- Under that condition factor prices are 'insensitive' to endowments, and FPE holds because free trade equalize goods prices. The diversification cone is found as in HOS and

$$p = A'\omega, c = H, F$$
$$V^c = Ay^c, c = H, F$$

Rybczynski in the 'Even' Case

- Differentiate the (FE) conditions w.r.t. the quantity of factor v, V_v:

$$\sum_{i=1}^{N} a_{vi}(\omega)\frac{dy_i}{dV_v} = 1$$

$$\forall v' \neq v, \sum_{i=1}^{N} a_{v'i}(\omega)\frac{dy_i}{dV_v} = 0$$

  or $A_{(V,N)}.[\frac{dy_i}{dV_v}]_{(N,V)} = Id_{(V,V)}$ in matrix form.
- If A is invertible, one can solve for $\frac{dy_i}{dV_v}$'s. In the first equation one $\frac{dy_i}{dV_v}$ must be positive. In the second equation one must be negative.

Theorem (Rybczynski)

A change in the endowment of each factor causes the output of one good to rise and that of another good to fall.

Stolper-Samuelson in the 'Even' Case

- Totally differentiating (ZP) yields:

$$\forall i, \hat{p}_i = \sum_{v=1}^{V} \theta_{vi}\hat{\omega}_v; \qquad \theta_{vi} \equiv \frac{\omega_v a_{vi}}{c_i}$$

- Suppose the price of just one good, i, increases.
- Then there exists at least one factor such that $\hat{\omega}_v \leq \hat{p}_i$ and another such that $\hat{\omega}_v \geq \hat{p}_i$.
- This is a weak Stolper-Samuelson result.

Are all factors vulnerable to falls in their real return?

- Recall from duality theory that $g_{pv} = g_{vp}$ characterize Stolper-Samuelson effects as well as Rybczynski effects.
- Solving for all $\frac{dy_i}{dV_v}$ is equivalent to solving for all $\frac{d\omega_v}{dp_i}$.

Theorem (Jones and Scheinkman, 1977)

Provided the A matrix is invertible at current factor prices, there exists for each factor a good such that an increase in its price lowers the real return of that factor.

- Proof: see Feenstra p70.
- Each factor has a good that is a 'natural enemy'. Trade liberalisation will require transfers.

More Goods Than Factors

In principle, when $N > V$

- (ZP) has more equations than unknowns, so there is no solution for factor prices except for special values of goods prices.
- (FE) has more unknowns than equations, so there are many solutions for the y_i's.
- we cannot predict Rybczynski effects because production y is indeterminate.

But it is possible to construct cases of FPE with special values of goods prices.

In a 3x2 example:

- only for special p_1, p_2, p_3 does (ZP) hold. Slightly different prices imply a corner solution where some y_i is zero.
- yet it is possible to build a FPE set where the IEE can be replicated
  - consider the IEE a_{vi}'s, and rank them by factor intensity
  - using goods demands D_i^w, plot factor demands $a_{vi} D_i^w$, starting from the origin and following that ranking
  - factor demands must sum up to world endowments as in the IEE, and form a FPE set
  - many combinations of output satisfy (FE) within that FPE set
  - when FPE holds the HOV result applies

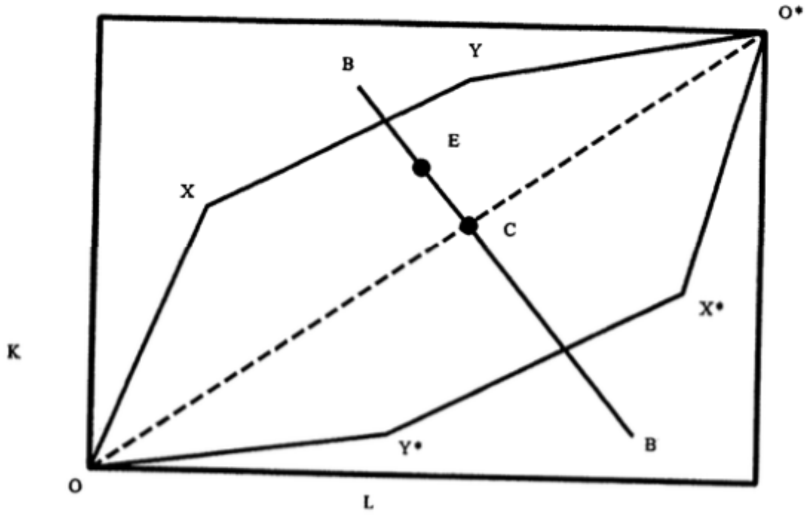


Figure: The FPE set in a HO model with 3 goods and 2 factors.

More Goods than Factors: DFS Model

- Outside the FPE set specialization occurs.
- We examine this in a special case: the Dornbusch, Fischer and Samuelson (1980) model with 2 factors and a continuum of goods.
  - Consider a continuum of goods indexed by $z \in [0, 1]$.
  - Production functions $y(z) = f_z[L(z), K(z)]$ and unit cost functions $c_z(w, r)$ are identical.
  - Assume no FIR and rank industries by *increasing* K-intensity $\frac{a_{Kz}(w,r)}{a_{Lz}(w,r)}$.
  - Assume identical Cobb-Douglas utility: $\ln(U) = \int_0^1 b(z) ln[c(z)]dz$.
- The FPE set is built as earlier, but has now a continuous shape.

More Goods than Factors: DFS Model

If FPE holds

- (FE) implies:

$$\frac{L^c}{K^c} = \frac{\int_0^1 a_{Lz}(w,r)y^c(z)dz}{\int_0^1 a_{Kz}(w,r)y^c(z)dz}$$

- (ZP) and the goods market-clearing condition imply:

$$\forall z, y^H(z) + y^F(z) = \frac{b(z)(wL^w + rK^w)}{c_z(w,r)}$$

- There are many y^H and y^F that satisfy these two equations. World production is determined, but domestic production is indeterminate.

Without FPE

- Each country specializes in goods for which its unit cost is lower than that of the other country.
- Suppose w.l.o.g. that $\frac{w^H}{r^H} < \frac{w^F}{r^F}$.
- (FE) requires that each country produces some goods, so there exists a threshold sector $\bar{z} \in (0, 1)$ such that $c_{\bar{z}}(w^H, r^H) = c_{\bar{z}}(w^F, r^F)$.
- Home has comparative advantage in $z < \bar{z}$ goods, Foreign has comparative advantage in $z > \bar{z}$ goods.

- The equilibrium vector $\{y, y^F, \bar{z}, w, r, w^F, r^F\}$ solves:

$$\forall z \in [0,\bar{z}], y^H(z) = \frac{b(z)(w^H L^H + r^H K^H + w^F L^F + r^F K^F)}{c_z(w^H, r^H)}$$

$$\forall z \in [\bar{z},1], y^F(z) = \frac{b(z)(w^H L^H + r^H K^H + w^F L^F + r^F K^F)}{c_z(w^F, r^F)}$$

$$\frac{L^H}{K^H} = \frac{\int_0^{\bar{z}} a_{Lz}(w^H, r^H) y^H(z) dz}{\int_0^{\bar{z}} a_{Kz}(w^H, r^H) y^H(z) dz}$$

$$\frac{L^F}{K^F} = \frac{\int_{\bar{z}}^1 a_{Lz}(w^F, r^F) y^F(z) dz}{\int_{\bar{z}}^1 a_{Kz}(w^F, r^F) y^F(z) dz}$$

$$\int_0^{\bar{z}} b(z)(w^F L^F + r^F K^F) dz = \int_{\bar{z}}^1 b(z)(w^H L^H + r^H K^H) dz$$

- Home specialises in $[0,\bar{z}]$, Foreign in $[\bar{z},1]$.
- As in HOV labor content is higher in Home than in Foreign exports.
- In addition, *every* good exported by Home has a higher labor content than every good exported by Foreign.

More Factors than Goods

With $V > N$:

- We can differentiate (ZP) as earlier, the weak Stolper-Samuelson result holds.
- But there are too many unknowns in (ZP), we cannot solve for factor prices. The Jones-Scheinkman theorem does not hold any more.
- FPE does not hold (or only for a set of measure zero).
- We cannot differentiate the (FE) conditions to get the Rybczynski result, because the a_{vi}'s depend on factor prices.
- We can derive some results in an interesting special case: the 2x3x2 Ricardo-Viner or 'specific factors' model (Jones, 1971).

The Ricardo-Viner (Specific Factors) Model

- Assumptions:
  - 2 sectors ($i = 1, 2$), 3 factors (L, K_1, K_2), 2 countries (H, F)
  - K_1 and K_2 are sector-specific, only L is mobile between sectors
  - CRS, perfectly competitive factor and goods markets
- Using the GDP function approach:

$$g(p, v) = \max_{L_1, L_2} \{ \sum_i^2 p_i f_i(K_i, L_i) \} \text{ s.t. } L_1 + L_2 = L$$

- GDP maximization implies

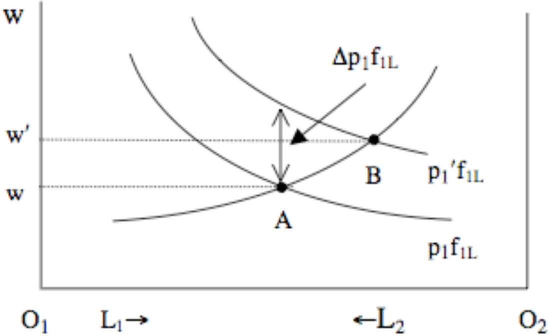
$$p_1 \frac{\partial f_1(K_1, L_1)}{\partial L_1} = p_2 \frac{\partial f_2(K_2, L_2)}{\partial L_2} = w$$

$$p_i \frac{\partial f_i(K_i, L_i)}{\partial K_i} = r_i, i = 1, 2$$

$$L_1 + L_2 = L$$

Stolper-Samuelson Effects in the Ricardo-Viner Model

- Wage response to a rise in p_1:



- The (ZP) curves depend on K_i endowments: FPE does not hold.
- Differentiating (ZP) with respect to p_i, it can be shown that:

$$\hat{p}_1 > \hat{p}_2 \Rightarrow \hat{r}_2 < \hat{p}_2 < \hat{w} < \hat{p}_1 < \hat{r}_1$$

The real returns to specific factors follow Stolper-Samuelson, but not real wages. The change in workers' welfare is ambiguous.

Rybczynski Effects in the Ricardo-Viner Model

- An increase in a specific factor's endowment reallocates labor towards that sector and away from the other sector.
  Graphically the $p_i \frac{\partial f_i}{\partial L}$ schedule is shifted outwards.
- An increase in the labor endowment causes the wage to fall, and *both* sectors to expand. There is no Rybczynski effect for labor.

Conclusions

- In the 'even' case:
  - a generalized no-FIR condition guarantees the existence of a FPE set
  - the Jones-Scheinkman theorem and a weak Rybczynski theorem apply
  - factor contents of net exports follow the HOV theorem
- With more goods than factors:
  - one can build a FPE set but production is indeterminate
  - inside the FPE set the HOV theorem applies
  - outside the FPE set specialization occurs and production also follows factor abundance.
- With more factors than goods:
  - FPE does not hold, we cannot solve for factor prices or Rybczinski effects
  - In the Ricardo-Viner model, there are S-S and Rybczynski effects for specific factors, but not labor